

On Real Options and Information Costs

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ABSTRACT

This paper presents a simple framework for the use of traditional capital budgeting models and the valuation of several real options in the presence of shadow costs of incomplete information. Information costs can be viewed as sunk costs in the spirit of Merton's (1987) model of capital market equilibrium with incomplete information. We incorporate these sunk costs in standard discounted cash flow techniques and present the basic concepts of real options. The justification of information costs in real projects is based on the observation that R&D needs to be done before investment decisions. These costs account for all the expenses needed to get informed about an investment opportunity and the management of projects. This analysis extends the models in Bellalah (1999, 2001) for the valuation of real options within information uncertainty. We present valuation models and simulations for the values of common real options in the presence of shadow costs of incomplete information.

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Key Words: Asset pricing; Option pricing; Information and market efficiency; Capital budgeting; Investment policy.

A company's value creation is determined by resource allocation and the proper evaluation of investment alternatives. Managers make capital investments to create future growth for shareholders. Investments lead to patents or technologies, which open up new growth possibilities. In general, managers use the basic investment techniques as the capital asset pricing model (*CAPM*), the cost of capital and the discount cash flow techniques, *DCF*. In investments valuation, organisations use also quantitative approaches such as net present value (*NPV*), decision tree analysis (*DTA*), payback time, or scenario/simulation which do not account for intangible factors such as future competitive advantage, future opportunities, managerial flexibility, the strategic value of an investment, etc. This is because the expected outcomes are not easy to forecast and the variability of investment returns may be extremely high. New techniques for capital budgeting incorporate real options, active management and strategic interactions between investment and financing decisions.³

Information plays a central role in the capital budgeting process and in investment and financing decisions. Edwards and Wagner (1999) study the role of information in capturing the research advantage and how to incorporate information into the decision process of active investment management. They show that implementation costs make sense only when weighted against the benefit of enhanced performance. They recognise that the most valuable commodity in the market is information that reduces uncertainty. In this spirit, trading cost information is part of the research that gives a manager active advantage. Edwards and Wagner (1999) show that managers must measure and develop confidence in the value of their research and then incorporate feedback from the market.

Merton (1987) adopts most of the assumptions of the original *CAPM* and relaxes the assumption of equal information across investors. He assumes that investors hold only securities of which they are aware. In his model, the expected returns increase with systematic risk, firm-specific risk, and relative market value. The expected returns decrease with relative size of the firm's investor base, referred to in Merton's model as the "degree of investor recognition". The intuition behind Merton's model is that investors consider only a part of the opportunity set and that full diversification is not possible, and that firm specific risk is priced in equilibrium. The main distinction between Merton's model and the standard *CAPM* is that investors invest only in the securities about which they are "aware". This

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³ For a survey of these techniques, the reader can refer to Brealey and Myers (1985), Copeland and Weston (1988), Smith and Nau (1994), Lee (1988), Agmon (1991) among others.

assumption is referred to as incomplete information. However, the more general implication is that securities markets are segmented. The intuition behind this result is that the absence of a firm-specific risk component in the *CAPM* comes about because such risk can be eliminated (through diversification) and is not priced. It appears from Merton's model that the effect of incomplete information on expected returns is greater than the highest specific risk of the firm and the highest weight of the asset in the investor's portfolio. The effect of Merton's non-market risk factors on expected returns depend on whether the asset is widely held or not.⁴

Kadlec and McConnell (1994) document the effect of share value on the NYSE and report the results of a joint test of Merton's (1987) investor recognition factor and Amihud and Mendelson's (1986) liquidity factor as explanations of the listing effect. The cross-sectional regressions provide support for investor recognition as a source of value from exchange listing. The regressions support Merton's model. The results also provide support for superior liquidity as a source of value from exchange listing. They provide also support to Amihud and Mendelson (1986) model.

Foerster and Karolyi (1999) construct an empirical proxy for the shadow cost of incomplete information for each firm, using the methodology in Kadlec and McConnell (1994). The investor recognition hypothesis of Merton suggests that abnormal returns may be due to the changes in the shareholder base, adjusted by the stock's residual variance and relative size. The results obtained by Foerster and Karolyi (1999) are supportive of the Merton (1987) hypothesis and consistent with Kadlec and McConnell (1994).

Coval and Moskowitz (1999) document the economic significance of geography and attempt to uncover the effect of distance on portfolio choice. They find that local equity preference is strongly related to firm size, leverage and output tradability. Their results suggest an information-based explanation for local equity. This is consistent with the findings in Kang and Stulz (1997) who find that foreign investors underweight small, highly levered firms, and firms that do not have significant exports. These results may be a response to severe information asymmetries associated with these firms.

Brennan and Cao (1997) develop a model of international equity portfolio investment flows which is based on the differences in informational endowments between foreign and domestic investors. The authors show that when domestic investors possess a cumulative information advantage over foreign investors about their domestic market, investors tend to purchase (sell) foreign assets in periods when the return on foreign assets is high (low).

Stulz (1999) examines the effect of globalisation on the cost of equity capital and argues that this cost decreases because of globalisation. The empirical evidence gives support to the theoretical prediction that globalisation decreases the cost of capital. He gives strong theoretical arguments justifying why the cost of capital should fall when markets become more open to foreign investors. Following Merton (1987), Stulz (1999) assumes that some investors do not hold some securities because they do not know about them. He provides a model in which this assumption amounts to attributing the home bias to ignorance or a non-modelled behavioural bias. This leads Stulz (1999) to show that the impact of globalisation on the cost of capital depends heavily on the extent of the home bias. However, the empirical evidence in Stulz (1999) shows that the effect of globalisation on the cost of capital is rather small because of the home bias.

Merton's (1987) model shows that asset returns are an increasing function of their beta risk, residual risk, and a decreasing function of the available information for these assets. Amihud and Mendelson (1988) consider several observed corporate policies that can be viewed as increasing the liquidity of investments. Their suggested policies include going public, instituting limited liabilities on equity

⁴ Merton's model may be stated as follows:

$$R_V - r = \beta_V [R_M - r] + \lambda_V - \beta_V \lambda_m$$

where:

R_V : the equilibrium expected return on an asset V ,

R_M : the equilibrium expected return on the market portfolio,

r : one plus the riskless rate of interest,

$\beta_V = cov(R_V/R_M)/var(R_M)$,

λ_V : the equilibrium aggregate "shadow cost" for the asset V . It is of the same dimension as the expected rate of return on this asset V ,

λ_m : the weighted average shadow cost of incomplete information over all assets.

claims, listing on organised exchanges, distributing ownership among many shareholders, etc. Since the transmission of this information is costly as in Merton's model, Amihud and Mendelson (1988) show how managers can balance the costs against the added value from the higher liquidity of the claims of the firm.

The above literature reveals the importance of information costs in the pricing of financial and real assets. Using this framework, Bellalah and Jacquillat (1995) and Bellalah (1999) develop simple models for the pricing of financial options in the presence of information costs. A similar analysis can be extended to real options using the same methods as in Bellalah (2001).

This work extends the standard capital budgeting techniques by accounting for the dynamic dimension of existing theories. The main objective is to analyse numerically the real option approach in capital budgeting investment decisions and compare this approach to the traditional *NPV*. This limits the study to only one stochastic underlying variable: the cash inflows.⁵

This paper is organised as follows. Section 1 reminds the use of traditional capital budgeting models. It incorporates also information costs in standard discounted cash flow techniques. Section 2 presents the basic concepts and specific features of real options. It develops also the general context for the pricing of options in the presence of information costs. Two cases are analysed: the case when the underlying asset is observable and the case when it is not observable nor continuously traded. Section 3 develops several models for the pricing of real options in the presence of information costs. Simulation results are proposed to show the impact of information costs on real option values.

1. TRADITIONAL CAPITAL BUDGETING MODELS AND INFORMATION COSTS

Investment decisions are often made with reference to standard discounted cash flow techniques, (*DCF* analysis). The most common capital budgeting models used by corporations involve either the basic net present value (*NPV*), Scenario/Simulation, or Decision Tree Analysis (*DTA*).

Basic *NPV* model:

The *NPV* is the sum of the expected future cash flows minus the initial costs of investments. This method seems to give better results than the accounting rate of return (*ARR*), the profitability index (*PI*), the internal rate of return (*IRR*), the modified internal rate of return (*MIRR*), and the payback method. However, this method ignores flexibility, assumes that the investment either falls into an reversible or an irreversible category, and that managers are given unbiased expected cash flows. For ease of exposition, the following notations are used.

$E_P(CF_t)$: expected cash flow,

R : risk adjusted discount rate,

r : risk-free discount rate,

\overline{CF}_t : certainty equivalent cash flow,

I_0 : investment outlay at time 0,

T : time to maturity,

$\lambda_s, (\lambda_c)$: information cost regarding the firm's cash flows (and the real option).

In the presence of information costs, the *NPV* can be written as:

$$NPV = \sum_{t=1}^T \frac{E_P(CF_t)}{(1+R+\lambda_s)^t} - I_0 = \sum_{t=1}^T \frac{\overline{CF}_t}{(1+r+\lambda_s)^t} - I_0$$

It is important to note that the information cost appears as an additional discount rate in the discounting of risky streams. This is the main intuition in Merton's (1987) model. In fact, this cost reflects the additional return required by investors to get compensated for their investments in information. An investor does not invest in a real project if he does not know about that project. The process of information acquisition has a cost that must be accounted for in the computation of the present value of cash flows. If the manager pays 2 millions in the process of information acquisition and the investment is equal to 100 million, than he must require at least 2/100 or 2% as an additional return

⁵ For a survey of the literature on real options, the reader can refer to Trigeorgis (1990, 1993, 1996), Pindyck (1991), Padock, Siegel and Smith (1987, 1988), Newton (1996), Myers (1984), Myers and Majd (1990) among others.

above the rate r . Hence, instead of a discount rate r , a new discount rate equal to $(r + 2\%)$ must be used as a rough approximation in this case.

Sensitivity Analysis:

Several managers rely on scenario analysis using high, low, or medium scenarios to bound the uncertainty. This method tends to show the impact on NPV and its sensitivity to each variable. Then the resulting NPV values are recorded. It assumes that other variables are constant in scenario base of their expected values. This technique recognises the existence of uncertainty but does not capture the flexibility due to “uncertainty” and offers little managerial guidance in investment decision process. In this analysis, information costs can be easily introduced in the simulation of the present values of risky streams in the same way as we have done for the calculation of the NPV .

Monte Carlo Simulation:

This method is not biased when modelling cash flows and deciding on the values for the relevant variables and correlation. For each variable, a probability distribution is designated and the cash flows are simulated discretely. Then, they are used to calculate the NPV . However, the serial dependency is complex to quantify. The NPV distribution given by the simulation is also hard to interpret economically⁶. This method is useful in the calculation of projects under uncertainty, even though, it has its proper limits. Information costs can also be easily integrated in this analysis in the discounting of the risky streams.

Decision Tree Analysis (DTA):

DTA approach takes into account later decisions and incorporates some of the managers flexibility into the valuation process. Investments are divided into a series of sub-investments that will be undertaken at different stages. The implementation of these investments in the future will depend upon some future event, thus enabling managers to decide whether to invest further or not. This process can not be implemented without additional information. This leads necessarily to information costs in the spirit of Merton (1987).

2. REAL OPTIONS: ANALYSIS AND VALUATION IN THE PRESENCE OF INCOMPLETE INFORMATION

During the last decade real options have been given an increasing interest by corporate practitioners in industries where the projects are costly and uncertain. Companies allocate resources for existing businesses or new ventures, and managers decide whether to invest now, to do nothing or to wait. When valuing investment decisions, the options to abandon or to defer, the options to expand or to switch are embedded into the project. These implicit options occur naturally or may be planned at some flexibility.⁷

Investment decision-making seems to be justified as a way to account for flexibility and can be thought of in terms of real options.⁸ Option pricing theory evaluates the firm as its operating options were managed optimally, without future information on optimal choices to be made. A distinction must be made between real assets, (which have a market value) and real options, (which consider the opportunities to purchase future real assets on favourable terms). Myers (1977) shows that the value of a firm is the combined value of the assets already in use and the present value of the future investment opportunities.

2.1. ANALYSIS OF REAL OPTIONS

Why real options are important ?

Investment is defined in financial economics as the act of incurring an immediate cost in the expectation of future rewards.⁹ The initial outlay is a payment for a right with no obligation to undertake a project. Real options give the right to receive a future cash flow from the investment cost. This is equivalent to a standard call option on a real asset.

⁶ Trigeorgis (1996).

⁷ These options appear in the work of Dentskevich and Salkin (1991), Dixit (1992, 1995), Dixit and Pindyck (1993, 1994, 1995), Faulkner (1996) and Ingersoll and Ross (1992) among others.

⁸ Dixit and Pindyck (1994).

⁹ Dixit and Pindyck (1995).

Using the option theory, the company can be viewed as a future possibility where an investor pays a premium for the right to buy a specific stock to a known exercise price at a certain time in the future. The investment amount is then the strike price, allowing the investor to capture the value of the underlying project.¹⁰ A real option strategy forces managers to compare every opportunity arising from existing investments with the full range of opportunities open to them. It promotes strategic leverage and encourages managers to exploit situations where investment can keep their company in the game. The strategy reduces the upside as well as the downside risk, and empowers managers to defer the investment opportunity without increasing the exercise price.

Difference between *NPV* and real options:

Real options can be used by managers with a basic understanding of option pricing models and tools. As they are important in strategic and financial analysis, they can be a complement to the standard *NPV* valuation. The *NPV* ignores the value of flexibility and creates a static picture of existing investments and opportunities. The traditional techniques treat opportunities as a “*now or never*” investment even if many investments can be deferred in the future without losing their value.

Strategic value of real options:

There is a large scope for applications of option pricing techniques for valuation of an entire firm.¹¹ A real option confers *flexibilities* to its holder as the option to invest, to wait, to divest, etc. These options can be economically important. The decision about when to invest is analogous to the decision about when to exercise an American call. The sensitivity of the value of the firm to these possibilities makes a real option valuation method better than the standard *NPV*. This is because an ordinary *NPV* valuation predicts future cash flows according to today’s information. By using the real option’s approach, the value of a company corresponds to the value of a portfolio of operating options yielding a stream of future cash flows. This portfolio can be seen as a portfolio of financial options on those future cash flows. Two types of flexibility are present in the project: internal and external flexibility.

Internal flexibility: corresponds to the managers flexibility to modify the project. This can include expansion, alteration, abandonment, etc.

External flexibility: corresponds to the growth option which gives the possibility to perform another project.

There are totally irreversible investments (where the whole investment cost is lost at the end of the operating phase), and partially irreversible investments (whose value can be partially recovered). Irreversibility can also arise from government regulations which makes investments irreversible. An irreversible investment opportunity is like a standard call option even if the asset can be sold to another investor.¹²

Two types of uncertainty are present in capital investments: economic uncertainty and technical uncertainty each with a positive increase effect on the value of a real option.

Economic uncertainty: is correlated with the actual exogenous movement of the economy: interest rate, inflation, industry prices, etc. This uncertainty could be reduced by waiting for new information before making the final investment.

Technical uncertainty: is the uncertainty in the project itself. It is endogenous to the decision process and is affected by management. For example, the uncertainty in the outcome of a R&D project can only be reduced with an actual step by step investment, until the future technical uncertainty is resolved.¹³

Analogy between financial and real options:

The analogy between financial and real options also has its limitations. There are three factors that make a real option different from a financial option:¹⁴ the proprietary state, the complex characteristics, and non tradability of real options. In fact, all financial options are proprietary and the holder decides when the option should be exercised. Real options present a proprietary characteristic, when the company has

¹⁰ Trigeorgis (1996).

¹¹ A typical example is firms in the oil and gas exploration and production business. Other examples include power stations and pharmaceutical companies. See for example, Paddock, Siegel and Smith (1988).

¹² Dixit and Pindyck (1994).

¹³ Dixit and Pindyck (1994).

¹⁴ Kester (1993).

a unique and exclusive know how in a technological process or has access to a patent. In general, investment opportunities with barriers to entry serve as proprietary real options. This is not the case when investment opportunities are shared by competitors and other participants.

When compared to the financial options markets, the real options markets are imperfect and only some proprietary real options can be traded with high transaction costs and few participants.¹⁵ Shared real options cannot be tradable on the market since they are already a public good for the whole industry.

Besides, most of financial options are derived from the underlying asset. Some real options have more complex characteristics. They give the holder the right not only to receive the gross present value of the future cash flows from the investment, but also investment opportunities in the future. In this case, the option becomes compounded and written on many another options.

Real options can be divided into two types: flexibility options and growth options. Growth options provide the firm with new opportunities down the line to undertake profitable follow-on investments.

Table 1: Summary of main real Options

Category	Description
Option to defer	In most investments opportunities, management holds an option to defer the life time of investment and see if the cash outflow meet the product price.
Time to build option	Managerial flexibility is embedded into the projects and can be valued as a compound option.
Option to expand	The management can expand the project if economic or technical conditions are favourable.
Option to abandon	Management can abandon current project and resale value of capital equipment.
Option to switch	Management can change the product flexibility by changing types of inputs.
Growth options	In general, investment is a link of interrelated projects opening future growth opportunities.

These real options are studied in different contexts by Kogut (1991), Kogut and Kulatilaka (1994a, b), Mac Donald and Siegel (1984, 1986), Brennan and Schwartz (1985), Berger, Ofek and Swary (1996) among others. Several other real options exist, but we restrict our analysis to these options. The same analysis applies to other options.

2.2. A GENERAL DERIVATION OF THE VALUES OF REAL OPTIONS

2.2.1. The valuation of options when the underlying asset is observable under incomplete information

Consider the following dynamics of the project's value:

$$dV/V = \mu dt + \sigma dz$$

where μ and σ refer to the instantaneous rate of return and the standard deviation of the project, and dz is a geometric Brownian motion. Let X be the price of a dynamic portfolio of assets perfectly correlated with V :

$$dX/X = \alpha dt + \sigma dz$$

where α stands for the expected return from owning a completed project.

Let $di = \alpha - \mu$. In this context, δ represents an opportunity cost of delaying investment. If di is zero, then there is no opportunity cost to keeping the option alive. Hence, the value of di must be positive. Let $G(V)$ be the value of the firm's option to invest. Using Merton's (1987) model, Bellalah and Jacquillat (1995) and Bellalah (1999, 2001) obtain option prices in the context of incomplete information.

Consider a portfolio: long an option which is worth $G(V)$ and go short G_V units of the project. The value of this portfolio is:

$$P = G - G_V V$$

Since the short position includes G_V units of the project, it requires the paying out of an amount $di V G_V$. The total return for this portfolio over a short interval of time dt is:

¹⁵ Trigeorgis (1996).

$$dG - G_V dV - di V G_V dt$$

Since there are information costs embedded in the option and its underlying assets, the return must be equal to $(r + \lambda_V)$ for the project and $(r + \lambda_C)$ for the option where λ_V and λ_C refer respectively to the information costs on the project and the option. In this context:

$$dG - G_V dV - di V G_V dt = (r + \lambda_C) G dt + (r + \lambda_V) V G_V dt$$

Assuming that a hedged position is constructed and since the application of Itô's lemma, we have:

$$dG = 1/2 G_{VV} (dV)^2 + G_V dV$$

we therefore have:

$$dG = 1/2 G_{VV} \sigma^2 V^2 dt + G_V dV$$

the value of dG is:

$$dG = 1/2 G_{VV} \sigma^2 V^2 dt + (\mu - di) G_V V dt + G_V V \sigma dz$$

We get after simplification:

$$1/2 G_{VV} \sigma^2 V^2 + (r + \lambda_V - di) V G_V - (r + \lambda_C) G = 0$$

When the time to maturity of the option is finite, this equation becomes:

$$1/2 G_{VV} \sigma^2 V^2 + (r + \lambda_V - di) V G_V - (r + \lambda_C) G + G_t = 0$$

For the valuation of standard calls, under the following condition:

$$G = \max(V - I, 0)$$

The call value is given by:

$$G = V \exp((\lambda_V - \lambda_C) T) N(d_1) - I \exp(- (r + \lambda_C) T) N(d_2) \quad (1)$$

$$d_1 = [\ln(V/I) + (r + \lambda_V + 1/2 \sigma^2) T] / \sigma \sqrt{T}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

2.2.2. The valuation of real options when the underlying asset is not observable nor continuously traded under incomplete information

Using the same analysis as in Merton (1998) and following the same approach as above, the equivalent of equation (28) in Merton (1998) is:

$$1/2 G_{VV} v^2 V^2 + (r + \lambda_V - di) V G_V - (r + \lambda_C) G + G_t = 0$$

where v^2 is the variance of the V-Fund portfolio in Merton (1998).

This equation can be solved under the following condition:

$$G(V, T) = E[h(VY)]$$

where Y is a log-normally distributed random variable with $E(Y) = 1$ and variance of $\ln(Y)$ is equal to $\theta^2 T$ and $E(\cdot)$ is the expectation operator over the distribution of Y .

The solution to this equation when:

$$h(V) = \max(V - I, 0)$$

is given by:

$$G = V \exp((\lambda_V - \lambda_C) T) N(d_{11}) - I \exp(- (r + \lambda_C) T) N(d_{11} - \sqrt{\gamma}) \quad (2)$$

$$d_{11} = [\ln(V/I) + (r + \lambda_V) T + \gamma/2] / \sqrt{\gamma}$$

$$\gamma = v^2 T + \theta^2 T$$

When compared to formula (1), this formula allows to understand the effect of the underlying asset price not being observable.

The main difference in the option pricing formula with and without continuous observation of the underlying asset is that the variance of the underlying does not go to zero around the maturity date because of the “jump” event at expiration. This formula can be applied when the underlying asset is neither continuously traded nor continuously observable.

This is a simple generalization of formula (27) in Merton (1998) to account for the effects of incomplete information.

3. REAL OPTIONS: VALUATION AND SIMULATION IN THE PRESENCE OF INCOMPLETE INFORMATION

The use of option valuation techniques in the valuation of real assets is based on some important assumptions.¹⁶ In general, individual values of real options are non-additive and the combined value could be complex to compute. Kulatilaka (1993) shows that the combined value of interacting options could either be higher or lower than the sum of the individual values. The combined value is dependent on the type of options, the degree of separation, the degree of being “in the money”, and the order of the options involved. Trigeorgis (1996) describes the interaction between options as basically additive. This is the case when the interacting options are of different types, i.e. calls and puts. He gives an example on the interaction between the option to abandon (which is equivalent to a put) and the growth option (which is equivalent to a call). He shows that these two options are additive because they are of different types.

3.1. The valuation procedure in the presence of information costs in a continuous-time setting

The valuation of financial options is based on the fact that an option can be replicated by a portfolio of traded securities. Since this equivalence is not dependent on risk attitudes, the value of the expected future payoffs can be derived from a risk-neutral approach and discounted at the risk-free interest rate. This concept can also be applied to real options, even if they are not traded in financial markets. The fundamental assumption is that a non traded project has the value that it would have had if it were traded in the financial markets.¹⁷

Trigeorgis (1996) shows that in the *DCF* analysis, the discount rate is received by identifying a twin security for each project. The twin security has the same risk characteristics as the specific project and is traded in financial markets. In this context, the option analogy could use the same twin security to replicate a no-arbitrage portfolio. Given the price of the project’s twin security, management can, in principle, replicate the returns to a real option by purchasing a certain number of shares while financing the purchase partly by borrowing at the risk-free rate. This makes possible the application of risk neutral valuation techniques for traded and non traded assets. The derivation of the standard formulas for option pricing in the presence of information costs appears in Bellalah (1999, 2001).

Gross present value of the project is the value of the expected cash flows to be received from the investment. It is considered significant without the investments. A higher present value of expected operating cash inflows can be achieved by increasing revenues, raising the price earned, producing more, or by generating compound business opportunities. The economic uncertainty is assumed to influence the gross present value and thus make it follow a geometric Brownian motion with a random part determined by the standard Wiener process $dz(t)$.

$$\frac{dV(t)}{V} = \mu dt + \sigma dz(t)$$

Where V refers to the gross present value of the cash flows, μ is the required rate of return and σ is the constant volatility.

Equilibrium requires that the total expected return to be the sum of expected capital gain plus the expected dividend d , so that $\mu = r + \lambda_s - di$. The stochastic equation can be written in a risk neutral world as:

$$\frac{dV(t)}{V} = (r + \lambda_s - di)dt + \sigma dz(t)$$

The capital investments to be made is the present value of the fixed costs over the lifetime of the investment. It is equivalent to the exercise price of a financial option. Here, we suppose certain capital investments. The reduce of the expected operating cash outflows can be achieved by leveraging economies of scale or by leveraging economies of scope in partnership.

¹⁶ For a survey of the literature on standard options and exotic options pricing, the reader can refer to Cox and Rubinstein (1985), Cox, Ross and Rubinstein (1979), Cox and Ross (1976), Black and Scholes (1973), among others.

¹⁷ Smith and Nau (1994).

The dividends¹⁸ are sums paid regularly to stockholders. This could be the costs incurred to preserve the option by keeping the opportunity alive, or the cash flows lost to competitors that go ahead and invest in an other opportunity. The cost of waiting could be high if an early entrant were to seize the initiative. The dividends are correspondingly high, thus reducing the option value of waiting and the value lost to competitors can be reduced by discouraging them from exercising their options. This is the case for example in locking up key customers or lobbying for regulatory.

The risk-free interest rate corresponds to the interest rate for a risk-free bond with the same expiration date as the project. Expected increase in the interest rate raises the option value, despite its negative effect on *NPV* (reduces the *PV* of the exercise price). Dixit & Pindyck argue that the risk free interest rate is useful for three types of real economic problems.¹⁹

- In complete markets, by changing the probability measure, any stochastic process can be transformed to a risk-neutral one.
- Economic applications assume that firms are risk-neutral even when investors and stockholders are risk-averse.
- No correlation between the market portfolio and macroeconomic shocks.

The volatility is the standard deviation of the growth rate of the value of future cash inflows. This is perhaps the crucial difference from *NPV* analysis. When uncertainty of expected cash flows raises, it increases the value of flexibility. For a project it could be a little more complex to find the correct volatility when compared to financial options.

Time to maturity corresponds to the time left until the opportunity disappears. It depends on technology (products life cycle), competitive advantages (intensity of competition), and contracts (patents, leases, licences). The time to maturity, is subjectively defined by management as the time it takes for competitors to exploit the same opportunity.

Dixit and Pindyck (1994) explain that the time to maturity is defined by the expiration of the patent. After the expiration, the firm loses the opportunity to gain a competitive advantage due to the patent. An increase in the opportunity's time raises the option's value because it increases the total uncertainty. The company might be able to extend its option by, extending exclusive raw material supply contracts, locking up distribution channels, etc.

The information costs are the costs engaged by investors to get informed about the projects and their real options. We make a distinction between information costs related to the underlying project cash flows and information costs related to each implicit real option.

Input Variables:

Because the value of a real option is determined by six parameters, exploiting proactive flexibility becomes simply a question of pulling one or more parameters. If the extended binomial or the extended Black & Scholes (1973) model in the presence of shadow costs of incomplete information is used, six input parameters are required in the valuation of any option: changes in the duration, the risk-free interest rate, the annual cost (or value lost over the duration of the option), expected cash inflows and cash outflows, the level of uncertainty and information costs. The following variables are used in the simulations.

Table 2: Real options Inputs

Gross present value of the project	V
Initial investment	I
Annualised dividend yield (%)	di
Risk-free rate (%)	r
Annualised standard deviation (%)	σ
The life time of the project (yrs)	T
Information costs for the real option and the underlying project	λ

¹⁸ or the lost value in time.

¹⁹ Dixit, Pindyck and Sodal (1999) use an exogenous discount rate for incomplete markets analysis.

3.2. The value of the option to invest

The value of the option to invest under incomplete information can be computed using the following equation:

$$1/2G_{VV}\sigma^2V^2 + (r + \lambda_V - di)VG_V - (r + \lambda_C)G = 0$$

This equation for the value of $G(V)$ must satisfy the following conditions:

$$G(0) = 0, G(V^*) = V^* - I, G_V(V) = 1$$

The value V^* is the price at which it is optimal to invest. At that time, the firm receives the difference $V^* - I$. The solution to the differential equation given in Bellalah (2001) is:

$$G(V) = aV^\beta \quad (3)$$

where:

$$\beta = 1/2 - (r + \lambda_V - di)/\sigma^2 + \left(\left[(r + \lambda_V - di)/\sigma^2 - 1/2 \right]^2 + 2(r + \lambda_C)/\sigma^2 \right)^{0.5}$$

$$V^* = \beta I / (\beta - 1), a = (V^* - I) / (V^*)^\beta$$

Table 3: Investment opportunity value $G(V)$: the effect of volatility

This Table simulates the value of the investment opportunity $G(V)$, given by equation (3), as a function of the project value, V , in the presence of information costs, λ . r is the interest rate, di is the opportunity cost of delaying project or a constant payout rate, I denotes the cost of investment or investment expenditure, σ stands for the volatility, λ_C (respectively λ_V) represents the information cost related to $G(V)$ (respectively V). It is assumed that $r = 4\%$, $\delta = 6\%$, $I = 1$, $\lambda_C = 1\%$, and $\lambda_V = 2\%$.

V	G(V)		
	$\sigma = 0.2$	$\sigma = 0.3$	$\sigma = 0.4$
0.12	0.0023	0.0095	0.0197
0.24	0.0103	0.0301	0.0534
0.48	0.0462	0.0958	0.1445
0.6	0.0748	0.1390	0.1990
0.84	0.1546	0.2435	0.3226
0.96	0.2063	0.3043	0.3908
1.08	0.2660	0.3703	0.4628
1.12	0.2877	0.3934	0.4876
1.16	0.3104	0.4171	0.5128
1.20	0.3339	0.4413	0.5383
1.24	0.3584	0.4661	0.5643
1.28	0.3838	0.4915	0.5906
1.32	0.4102	0.5173	0.6173
1.44	0.4949	0.5981	0.6994
1.56	0.5883	0.6834	0.7846
1.68	0.6903	0.7733	0.8726
1.80	0.8012	0.8675	0.9635
1.92	0.9209	0.9661	1.0570
2.04	1.0498	1.0688	1.1531
2.16	1.1875	1.1756	1.2517

This Table simulates the value of the investment opportunity, $G(V)$. All things being equal, a larger volatility can be associated with a greater value of the option to invest. And the high project values generate an increase in the value of the option to invest.

3.3. The value of the option to defer

Some projects could increase in value when new information is available and uncertainty decreased with more favourable conditions. The value of waiting to invest or the option to defer can be seen as an American call option on the gross present value of the future expected cash flows.²⁰

The option to defer is reversible and more valuable when there is high economic uncertainty and long investment horizons.²¹ For simplicity, the value of this option can be simulated using the following formula.

$$G(V, I, r, di, T - t, \sigma, \lambda_S, \lambda_C) = \eta \left(V e^{-(\lambda_C - \lambda_S)(T-t)} N(\eta d_1) - I e^{-(\lambda_C + r)(T-t)} N\left(\eta \left(d_1 - \sigma \sqrt{T-t}\right)\right) \right) \quad (4)$$

²⁰ Trigeorgis (1996).

²¹ Ingersoll and Ross (1992).

With:

$$d_1 = \frac{\ln\left(\frac{V}{I}\right) + \left(r + \lambda_s - di + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

Where $\eta = 1$ for a call and -1 for a put.

3.4. The value of the time-to-build option

Few investments in practice are a single up-front outlay. However, most investments are sequential and staged into several investments. This creates valuable options to default at any given stage. The completion of one stage gives the right but not the obligation to undertake the next stage and the options that this stage provides. The staged investment can be viewed as a series of compound options. In this case, the valuation process can be computed discretely. The project in this case is a perpetual cash flow with a fixed capital outlay. There are points when the project has a positive *NPV*, but we are better off not taking it because the option to undertake the project in the future is more valuable. Since the investment is irreversible, when we take the project, we destroy the value of waiting. It is possible in this context to extend the standard binomial model to account for the effects of information costs. When generating the binomial tree for the underlying asset, we must account for the information cost of the asset. When we work backward, we must account for the information cost regarding the option.

The valuation procedure in a discrete-time setting in the presence of information costs:

The valuation procedure modifies slightly the binomial model to account for the effects of incomplete information. It can be described in the following steps.

- The gross present value is V
- The up multiplier (u) and down multiplier (d) are calculated by using formulas in Cox, Ross and Rubinstein (1979):

$$u = e^{\sigma\sqrt{h}}, d = e^{-\sigma\sqrt{h}}$$

with:

$$h = \frac{T-t}{N} \quad (5)$$

They are used to calculate the future gross value (V) in the nodes of the binomial tree.

- The risk neutral probability for the up and down branches are calculated as:

$$p = \frac{e^{(r-d+\lambda_s)h} - d}{u - d} \quad (6)$$

- The discount factor at each node is:

$$e^{-(r+\lambda_c)h} \quad (7)$$

- The binomial tree should be constructed in such way that it can incorporate the investments needed.
- Count backward from the end, in every node calculate the value by using the binomial formula for one period and subtracting the value of the investment. To consider this in the binomial tree, the value at each node should be the maximum of the value of the project in the node and zero.
- The calculation of the value in each node should continue in this backward calculation until the value of the firm finally reaches the present time.

The following Tables 4 and 5 and Figures 1, 2 and 3 simulate the values of the time to build option in the presence of information costs for the option and its underlying asset.

Table 4: Time to build option using binomial approach

Initial Cash flow (V)	300	h	1
Initial investment (I)	800	Up multiplier	1.49182
Number of years to Maturity(T)	6	Down multiplier	0.67032
σ	40%	Up probability	0.40131
r	10%	Down probability	0.59869
λ_s	0%	Discount Factor	0.90484
λ_C	0%		

Figure 1: Time to build binomial tree

							3306.95
							32269.5
						2216.72	
						21367.2	
						19333.8	
					1485.91	21367.2	1485.91
					14059.1		14059.1
					12721.2		
				996.035	14059.1	996.035	
				9160.35		9160.35	
				8288.63		8288.63	
		667.662		9160.35	667.662	9160.35	667.662
		5876.62			5876.62		5876.62
		5317.39			5317.39		
	447.547	5876.62	447.547	5876.62	447.547	447.547	
	3675.47		3675.47		3675.47	3675.47	
	3325.71		3325.71		3325.71	3325.71	
300	3675.47	300	3675.47	300	3675.47	3675.47	300
2200		2200		2200			2200
1990.64		1990.64		1990.64			
2200	201.096	2200	201.096	2200	201.096	201.096	
	1210.96		1210.96		1210.96	1210.96	
	1101.45		1095.72		1095.72	1095.72	
	1210.96	134.799	1210.96	134.799	1210.96	1210.96	134.799
		547.987		547.987		547.987	547.987
		558.558		495.839			
		558.558	90.3583	547.987	90.3583	90.3583	
			103.583		103.583	103.583	
			219.362		93.7254	93.7254	
			219.362	60.569	103.583	103.583	60.569
				-194.31			-194.31
				37.6132			
				37.6132			
					40.6006	40.6006	
					-393.99	-393.99	
					0	0	
					0	0	27.2154
							-527.85

Terminal column has two elements in each state:

- State variable
- *NPV*

Earlier columns have four elements in each state:

- State variable
- *NPV* (if project is undertaken)
- Option Value
- Project Value

Table 5: Time to build option using binomial approach

<i>V</i>	300	<i>h</i>	1
<i>I</i>	800	Up multiplier	1.49182
<i>T</i>	6	Down multiplier	0.67032
σ	40%	Up probability	0.40131
<i>r</i>	10%	Down probability	0.59869
λ_S	1%	Discount Factor	0.89583
λ_C	1%		

Figure 2: Time to build binomial price and standard NPV

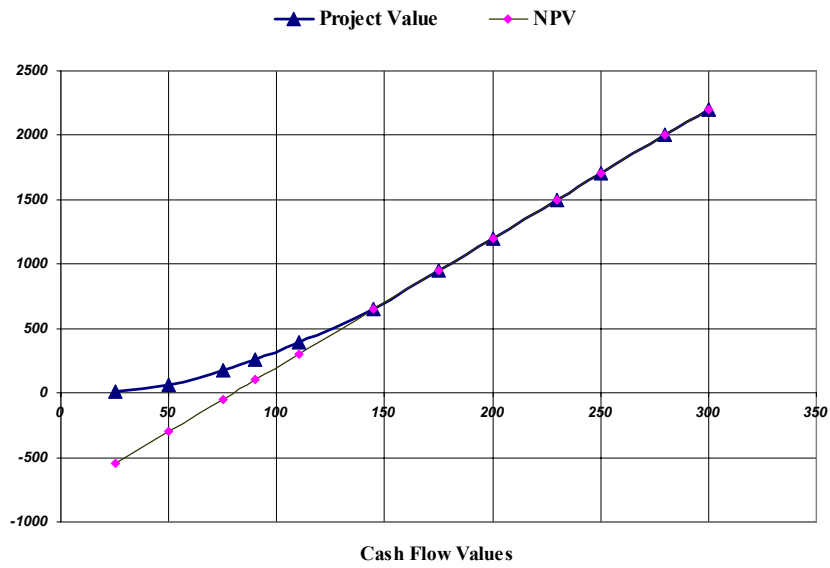


Figure 3: Time to build binomial price and standard NPV

						3511.44
						34314.4
					2353.79	
					22737.9	
					20369.4	
				1577.79	22737.9	1577.79
				14977.9		14977.9
				13417.7		
			1057.63	14977.9	1057.63	
			9776.26		9776.26	
			8757.91		8757.91	
		708.948	9776.26	708.948	9776.26	708.948
		6289.48		6289.48		6289.48
		5634.33		5634.33		
	475.222	6289.48	475.222	6289.48	475.222	
	3952.22		3952.22		3952.22	
	3540.54		3540.54		3540.54	
318.551	3952.22	318.551	3952.22	318.551	3952.22	318.551
2385.51		2385.51		2385.51		2385.51
2137.02		2137.02		2137.02		
2385.51	213.531	2385.51	213.531	2385.51	213.531	
	1335.31		1335.31		1335.31	
	1196.22		1196.22		1196.22	
	1335.31	143.134	1335.31	143.134	1335.31	143.134
		631.342		631.342		631.342
		618.278		565.577		
		631.342	95.9457	631.342	95.9457	
			159.457		159.457	
			257.719		142.847	
			257.719	64.3143	159.457	64.3143
				-156.86		-156.86
				57.3263		
				57.3263	43.1112	
					-368.89	
					0	
					0	28.8983
						-511.02

3.5. The value of the option to expand

An option to expand is a call option to acquire an additional part to the initial project where the cost to expand is the exercise price. This managerial flexibility has a value and the cost of expanding could be reduced if flexibility is built into the project at an early stage. The value of this option in the presence of shadow costs of incomplete information can be computed using the following formula.

$$G(V, I, r, di, T - t, \sigma, \lambda_s, \lambda_c) = \eta \left(V e^{-(\lambda_c - \lambda_s)(T-t)} N(\eta d_1) - I e^{-(\lambda_c + r)} N\left(\eta \left(d_1 - \sigma \sqrt{T-t}\right)\right) \right) \quad (8)$$

With:

$$d_1 = \frac{\ln\left(\frac{V}{I}\right) + \left(r + \lambda_s - di + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

Table 6: Time to expand option values using binomial approach

This Table simulates the value of the time to expand option in the presence of information costs for the option and its underlying asset. The values of calls and puts are presented to check the put call parity relationships. It is assumed that $I = 500$, $di = 6\%$, $r = 5.5\%$, $\sigma = 45\%$, and $T = 12$.

λ_s	0.00%	0.00%	1.00%	1.00%
λ_c	0.00%	1.00%	0.00%	1.00%
V	Call Values			
1	0.00008	0.00007	0.00013	0.00011
50	2.80938	2.49169	3.56334	3.16040
100	10.45256	9.27059	12.93768	11.47470
150	21.14861	18.75714	25.83219	22.91110
200	33.94988	30.11084	41.10238	36.45454
250	48.30553	42.84316	58.10098	51.53094
300	63.86224	56.64072	76.42005	67.77850
350	80.37755	71.28849	95.78343	84.95228
400	97.67687	86.63161	115.99436	102.87777
450	115.62982	102.55445	136.90720	121.42580
V	Put Values			
1	257.93900	228.77137	257.87698	228.71637
200	195.02510	172.97174	189.76572	168.30710
400	161.40163	143.15041	154.89537	137.37987
600	138.88397	123.17903	131.99991	117.07342
800	122.31150	108.48057	115.37312	102.32678
1000	109.43488	97.06003	102.58516	90.98487
1200	99.06303	87.86103	92.36933	81.92425
1400	90.48898	80.25653	83.98269	74.48596
1600	83.25972	73.84475	76.95371	68.25182
1800	77.06841	68.35355	70.96573	62.94096

This Table simulates the value of the time to expand option in the presence of information costs for the option and its underlying asset. The high project values generate an increase in the value of the option to expand. In the presence of the shadow costs of incomplete information regarding project value, the option value increases. In the case where information costs concern the option value, option to expand value drops instead of increasing. It is of interest to note that the negative effect due to incomplete option value information and the positive effect due to incomplete project value information are compensated. But, on the whole, the presence of two types of information costs increases the option to expand value compared to its level in the complete information case.

3.6. The value of the option to contract

The option to contract has a positive value if market conditions turn weaker than originally expected in this case, management can then reduce the scale of operations and thus saving part of the planned investment outlays. This analogous to a put option on part of the initial project, with exercise price equal

to the potential cost savings. This may be particularly valuable in the case of new-product introductions in uncertain markets.

$$G(V, I, r, di, T - t, \sigma, \lambda_S, \lambda_C) = \eta \left(V e^{-(\lambda_C - \lambda_S)(T-t)} N(\eta d_1) - I e^{-(\lambda_C + r)} N\left(\eta \left(d_1 - \sigma \sqrt{T-t}\right)\right) \right) \quad (9)$$

with:

$$d_1 = \frac{\ln\left(\frac{V}{I}\right) + \left(r + \lambda_S - di + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

Table 7: Option to contract values

This Table simulates the value of the option to contract in the presence of information costs for the option and its underlying asset. The opportunity to contract the scale of the project by 5%, saving an amount of 100. It is assumed that $di = 6\%$, $r = 5.5\%$, $\sigma = 45\%$, and $T = 12$.

λ_S	0.00%	0.00%	1.00%	1.00%
λ_C	0.00%	1.00%	0.00%	1.00%
V	Option Values			
1	51.66080	45.81902	51.65769	45.81626
50	50.49407	44.78422	50.34767	44.65438
100	49.38165	43.79760	49.11073	43.55731
150	48.34655	42.87954	47.96912	42.54479
200	47.37949	42.02183	46.90969	41.60516
250	46.47192	41.21689	45.92108	40.72834
300	45.61670	40.45838	44.99408	39.90617
350	44.80790	39.74104	44.12124	39.13203
400	44.04060	39.06051	43.29644	38.40050
450	43.31066	38.41311	42.51459	37.70706
500	42.61457	37.79573	41.77144	37.04794
550	41.94931	37.20570	41.06335	36.41993

This Table simulates the value of the option to contract in the presence of information costs for the option and its underlying asset. As expected, The high project values generate a decrease in the value of the option to contract : the option to contract has a positive value if market conditions are unfavourable.

3.7. The value of the option to shut down and restart operations

The managerial flexibility to be able to shut-down and restart operations can be valuable if prices are such that cash revenues are not sufficient to cover variable operating costs. It might be better not to operate temporarily. If prices rise sufficiently, operations can be restarted. Thus, operations in each year may be seen as a call option to acquire that year's cash revenues by paying the variable costs of operating as a strike price. It is equivalent to the firm having a portfolio of call and put options. For example, being able to temporarily shut down a project is equivalent to a put option and restarting operations when the project has been down is equivalent to a call option.

Why a company may choose to stay in a line of business (or stay in business, generally, even though it is currently running a loss and the NPV of future operations is negative. The intuition is that there are irreversible costs of exiting and re-entering, and if you exit now, you may wish in the future that you had not.

3.8. The value of the option to abandon

The option to abandon can be valued as an American put option on the project's current value, with an exercise price corresponding to the salvage or best alternative use value. If prices suffer a sustainable decline or the operation does poorly for some other reason, management may have a valuable option to abandon the project in exchange for its salvage value. The option to abandon a project provides partial insurance against failure.

$$G(V, I, r, di, T - t, \sigma, \lambda_S, \lambda_C) = \eta \left(V e^{-(\lambda_C - \lambda_S)(T-t)} N(\eta d_1) - I e^{-(\lambda_C + r)} N\left(\eta \left(d_1 - \sigma \sqrt{T-t}\right)\right) \right) \quad (10)$$

With:

$$d_1 = \frac{\ln\left(\frac{V}{I}\right) + \left(r - di + \lambda_s + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

Table 8: Option to abandon values

This Table simulates the values of the option to abandon for different levels of information costs. I is the value received on abandonment and T is the number of years until abandon (yrs). It is assumed that $I = 150$, $di = 5\%$, $r = 5\%$, $\sigma = 40\%$, and $T = 10$.

λ_s	0.00%	0.00%	1.00%	1.00%
λ_c	0.00%	1.00%	0.00%	1.00%
V	Call Values			
1	0.00007	0.00006	0.00010	0.00009
50	6.26693	5.67055	7.61406	7.61406
100	22.25988	20.14157	26.32698	22.25988
150	43.02524	38.93085	50.19539	43.02524
200	66.47903	60.15271	76.88814	66.47903
250	91.61911	82.90040	105.31919	91.61911
300	117.89443	106.67529	134.90372	117.89443
350	144.97336	131.17732	165.29530	144.97336
400	172.64304	156.21389	196.27438	172.64304
450	200.75999	181.65515	227.69450	200.75999
500	229.22371	207.41019	259.45405	229.22371
550	257.96164	233.41334	291.48020	257.96164
V	Put Values			
1	90.37313	81.77299	90.30938	81.71530
50	66.92000	60.55172	65.07766	65.07766
100	52.58641	47.58215	50.27457	52.58641
150	43.02524	38.93085	40.62699	43.02524
200	36.15249	32.71213	33.80373	36.15249
250	30.96604	28.01923	28.71878	30.96604
300	26.91483	24.35355	24.78730	26.91483
350	23.66723	21.41499	21.66288	23.66723
400	21.01038	19.01098	19.12596	21.01038
450	18.80079	17.01166	17.03008	18.80079
500	16.93798	15.32612	15.27362	16.93798
550	15.34938	13.88869	13.78377	15.34938

This Table simulates the values of the option to abandon for different levels of information costs and shows that the value of the option to abandon the project increases when market conditions decline severely (that is, when the value of the project is weak).

3.9. The value of the option to switch and the growth option

The firm should be willing to pay a certain positive premium for a flexible technology that can change the inputs from expensive to cheap and change the output from cheap to expensive, depending on the market. Process flexibility can be achieved not only via technology (e.g. by building a flexible facility that can switch among alternative energy inputs), but also by maintaining relationships with a variety of suppliers and switching among them as their relative prices change.

The growth option provides the company with a possibility to make a follow-on investment in the future, it is analogous to a call option. The option to grow is used when an initial investment is required for further development. The project can be considered as a link in a chain of related projects and may serve as a springboard for future project generations. But unless the firm makes that initial investment, subsequent generations will not be feasible.

Kester (1984) recognised the importance of the real growth option on firms and argued that the growth option constituted can account for more than half of the market value for most of the companies. The value of the growth option can be computed using the following formula.

$$G(V, I, r, di, T-t, \sigma, \lambda_s, \lambda_c) = \eta \left(V e^{-(\lambda_c - \lambda_s)(T-t)} N(\eta d_1) - I e^{-(\lambda_c + r)} N\left(\eta \left(d_1 - \sigma\sqrt{T-t}\right)\right) \right) \quad (11)$$

with:

$$d_1 = \frac{\ln\left(\frac{V}{I}\right) + \left(r + \lambda_S - di + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

Table 9: Growth option prices

This Table simulates the values of the growth option for different levels of information costs. I is the investment cost. It is assumed that $I = 30$, $di = 5\%$, $r = 7\%$, and $\sigma = 35\%$.

λ_S	0.00%	0.00%	1.00%	1.00%
λ_C	0.00%	1.00%	0.00%	1.00%
V	Call Values			
1	0.00950	0.00818	0.01411	0.01215
25	6.27361	5.39975	7.79132	6.70605
50	16.31713	14.04428	19.78988	17.03331
75	27.22501	23.43279	32.66339	28.11364
100	38.48884	33.12765	45.88367	39.49244
125	49.93540	42.97980	59.27752	51.02064
150	61.48820	52.92338	72.77038	62.63405
175	73.10801	62.92464	86.32475	74.30040
200	84.77264	72.96449	99.91972	86.00170
225	96.46860	83.03129	113.54277	97.72716
250	108.18723	93.11761	127.18593	109.46994
275	119.92274	103.21846	140.84393	121.22550
V	Put Values			
1	10.03527	8.63744	9.96343	8.57561
25	4.96258	4.27133	4.56916	3.93271
50	3.19693	2.75162	2.84744	2.45081
75	2.29565	1.97589	2.00065	1.72198
100	1.75032	1.50651	1.50064	1.29161
125	1.38772	1.19442	1.17420	1.01065
150	1.13135	0.97376	0.94677	0.81489
175	0.94199	0.81078	0.78084	0.67208
200	0.79746	0.68638	0.65553	0.56422
225	0.68426	0.58895	0.55828	0.48052
250	0.59373	0.51102	0.48115	0.41413
275	0.52007	0.44763	0.41887	0.36052

This Table simulates the values of the growth option for different levels of information costs. The high project values generate an increase in the value of the growth option. In the presence of the shadow costs of incomplete information regarding project value, the option value increases. In the case where information costs concern the option value, the growth option value drops instead of increasing. And the presence of two types of information costs increases the growth option value compared to its level in the complete information case.

Concluding Remarks

This paper reviews the main well known concepts in real options and extends the literature for the valuation of real options in the presence of information costs as in Bellalah (1999, 2001). These options are fundamental in the valuation process of investments and capital budgeting. However, they are valued in a standard framework ignoring the role of information costs in investment decisions. Information costs play a central role in the capital budgeting process since managers do not invest in projects they do not know about. When money is engaged in research and development, in project analysis and valuation, it is natural to require a return that accounts for these expenses. Therefore, information costs or shadow costs of incomplete information represent a component of the appropriate discount rate in investment decisions.

We introduce information costs in the spirit of Merton (1987), Bellalah (1990), Bellalah and Jacquillat (1995) and Bellalah (1999, 2001) in the capital budgeting process and real options valuation. We develop a general derivation for the valuation of options when the underlying asset is observable and when it is not observable. This provides a generalisation of the Black-Scholes (1973) formula and Merton (1998) formula which accounts for the effects of incomplete information. We present different

formulas for the valuation of the option to defer, the time to build option, the option to shut down and restart option, the option to abandon, the switch option and the growth option in the presence of information costs. Simulation results are provided using reasonable values for information costs. Our analysis can be extended to other types of real options. In particular, it can be applied to compound real options and “exotic” real options using the same techniques as those in the pricing of exotic options as shown in Bellalah (2000). The models can also be tested using real data. It is also possible to extend these models to account for stochastic volatility of the cash flows as in Aboura, Bellalah, Villa and Prigent (2000).

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