The Determinants of Default Correlations

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The Determinants of Default Correlations

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Abstract

This paper extends empirical work on default risk in three ways. First, it estimates expected default probabilities (using structural models) and compute default correlations (via a copula function) for a sample of US companies. Second, it extracts common or latent factors that drive companies’ default correlations using factor-analytical technique. The results indicate that only the common factors related with the overall state of economy explain default correlations. Third, since idiosyncratic risk has different implications for risk management, this study examines this important variable. Idiosyncratic risk does change significantly prior to bankruptcy, which suggests that financial markets also react to company specific signals.

JEL: C30; G13; G33.

Keywords: Expected Default Probabilities; Structural Models; Idiosyncratic Risk; Default Correlations

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1. Introduction

Corporate defaults exhibit two prominent characteristics that have unique implications for default risk management. First, default risk is correlated through time. Bankruptcies are normally the end of a process that begins with economic difficulties and end with financial distress. Although some bankruptcies are unexpected and, therefore, are point events, like Enron and Worldcom, investors become aware of company’s difficulties some years before bankruptcy occurs. Second, economic intuition suggests that financial wealth of companies in the same industry or within the same economic area is a function of managers’ skills and common factors that introduce correlations between them. While financial institutions, namely banks, are aware of these relationships, their ability to model such correlations is still not fully developed. Basle Committee on Banking and Supervision (BCBS, 1999, p. 31) states “... the factors affecting the credit worthiness of obligors sometimes behave in a related manner...” which “... requires consideration of the dependencies between the factors determining credit related losses”. Whilst there are several models and even approaches to compute default probabilities, there is no consensus on the importance of different factors that drive default correlations.

Companies’ financial and economic wealth are linked through sector-specific and/or macroeconomic factors. Whilst a great deal of effort has been made by practitioners to understand and measure default correlations among companies, only recently academics have displayed interest in this issue. Crouhy, Galai and Mark (2000) describe the most popular credit risk frameworks used and sold by financial institutions.
For example, CreditMetric and KMV\(^1\) (building on Merton (1974) model) consider that for each pair of obligors, their asset values follow a joint normal distribution. Under this assumption and according to Merton’s model, dependence between defaults is driven by the dependence between asset and threshold values and default correlation can be computed using traditional statistics techniques and bivariate normal distribution. In the actuarial CreditRisk\(^+\) \(^2\) model, default correlations are driven by common factors.

The efficacy of diversification within a portfolio of claims requires accurate estimates of correlations in credit events for all pairs of obligors. For example, Collateralised Bond Obligation (CBO) and valuation of credit derivatives, as discuss by Hull, Predescu and White (2005), require estimates of joint probability of default over different time periods and for all obligors. Default correlations can lead to a dramatic change on the tails of an investor’s portfolio probability density function of credit losses (PDCL) and consequently on the required economic capital for face unexpected losses. The common assumption in financial research of independence between events produces the right tail of the PDCL thinner than the one observed, which means that observed unexpected losses are higher than the ones estimated. BCBS (1999) points out that portfolios’ PDCL are skewed toward large losses and more difficult to model than the one of market risk. The PDCL that results from the combination of single credit exposures depends strongly on the assumptions about credit correlations.

This study is concerned with a component of credit correlations, namely, default correlations. The paper can be motivated in a number of ways. A number of authors

\(^1\) CreditMetrics was developed by RiskMetrics Group. KMV was developed by KMV Corporation.
\(^2\) CreditRisk\(^+\) was developed by Credit-Suisse Financial Products.
(Frey and McNeil (2001), Davis and Lo (2001)) have investigated copula function and the notion of extremal dependence of risk factors to model default correlations in the context of the management of a loans portfolio. Insofar as defaults are infectious, an analysis of default correlations is potentially important. Another line of investigation (Giesecke (2004)) has focused on the relationship between cyclical default correlations and macro-economic variables.

Our main objective is to identify the determinants of default correlations. We apply a set of structural models [Merton (M, 1974), Longstaff and Schwartz (LS, 1995) and Ericsson and Reneby (ER, 1998)] to compute companies’ expected default probabilities (EDPs) and, using factor-analytical techniques, extract the common or latent factors that explain default correlations. Giesecke (2004) points out that default correlations between companies is due to their dependence on macro-economic variables, which causes cyclical default correlation, and operational and financial relationships with other companies, which causes default contagion effects. In order to capture default contagion effects, we narrow our analysis to bond markets related variables. We examine the extent to which default correlation can be ascribed to each of these factors and the extent to which they can be explained by systematic variables (from capital and bond markets). We also examine the predictive power of idiosyncratic risk for bankruptcy. Given that unexpected events or fraudulent defaults lead to market-wide jumps in credit spreads, which reduce the ability to diversify this risk, it is also important to examine the relationship between company’s idiosyncratic risk and bankruptcy. To date, none of the default correlation analysis has been concerned with idiosyncratic risk, therefore, it is interesting to extend the work.
The results show that the set of structural models applied are able to predict bankruptcy events. Another important finding is the relevance of idiosyncratic risk (and not of total volatility) in predicting default events. This suggests that company specific signals provide useful information to investors about the deterioration in company’s economic and financial conditions prior to bankruptcy. Factor-analytical technique extracts factors that explain around 83 percent of the variability of default correlations. The determinants of these factors are variables that proxy the overall state of the economy and the expectations of its evolution.

The existing literature on default correlations can be divided into two approaches: the one resulting from the structural approach that models default correlations through companies’ assets values; and the one derived from the reduced-form approach that models default correlations through default intensities (see Duffie and Singleton (2000) for the simulation techniques).

Li (2000) is one of the first efforts to systematize default correlations. The author models default correlation between two companies as the correlation between their survival times. He uses the copula concept to define the joint distribution of survival times with given marginal distributions. Li (2000) shows that CreditMetrics uses a bivariate normal copula function with asset correlation as the correlation factor. Laurent and Gregory (2003) extend this work is extended to several obligors.

Hull and White (2000) develop a method to value vanilla credit default swap when there is counterparty default risk, which is modelled by the default correlation. The authors assume that dependence structure among companies’ defaults follows a
multivariate normal distribution. Hull et al. (2005) extend the previous model to several obligors. They assume that companies’ default threshold has a systematic and an idiosyncratic component. The systematic component is defined as the sensitivity of the threshold to a factor (systematic), common to all firms. Default correlation is defined as the product of each company loading to the systematic variable. Zhou (2001) provides an analytical formula to compute default correlations and joint default probability within first-passage models. However, the empirical application of this framework to portfolio of loans or bonds becomes cumbersome since it only supports two obligors at each time.

Frey and McNeil (2001) apply a copula function and the notion of extremal dependence of risk factors to model default correlations in the context of the management of a loans portfolio. Davis and Lo (2001) study how “infectious defaults” (or contagion effects) can be introduced within the Binomial Expansion Technique developed by Moody’s. The authors investigate this issue assuming that default correlation among all firms of a CBO is equal and time independent.

Schonbucher (2003) argues that default correlation spreads by other channels than business ties. Assuming an imperfect market, with asymmetric information, default contagion can arise from information effects, learning effects or updating of beliefs, which means that the default of one company provides information about the default risk of other companies. Collin-Dufresne, et. al (2003) study default contagion via updating of beliefs, within a reduced-form model. According to the authors, in the past, events like unexpected or fraudulent defaults lead to market-wide jumps in credit spreads, which reduce the ability to diversify this risk.
In Section 2, a brief digression on relevant dependence measures, with an exposition of copula functions, presented. Section 3 outlines the economic intuition behind this research. Section 4 contains empirical work. Section 5 discusses the implications of the results and section 6 reports the conclusions.

2. Brief Digression by Dependence Measures

Pearson correlation coefficient, $\rho$, commonly used in finance as a measure of dependence between two variables, assumes that financial data follows a multivariate normal distribution, which means that it can only be used in the elliptical world (see Embrechtz, McNeil and Straumann (2001) to limitations about this measure). However, financial data probability distribution is not normal, has fat tails and skewness (this subject is particularly important in credit risk management), meaning that there are other dimensions of risk that must be consider. One of these dimensions is the dependence structure between the variables. The copula function allows us to measure this dimension.

In this section, we do a brief review of copula concepts. A full understanding of this subject can be done in Frees and Valdez (1998), Nelsen (1999) and Costinot, Roncalli and Teiletche (2000).

A copula function defines the dependence structure between random variables. It links univariate marginals to their multivariate distribution. Consider $p$ uniform random
variables, \( u_1, u_2, ..., u_p \). The joint distribution function among these variables is defined as

\[
C(u_1, u_2, ..., u_p) = \text{Prob} \{ U_1 \leq u_1, U_2 \leq u_2, ..., U_p \leq u_p \}
\]

where \( C \) is the copula function.

Copula functions are also used to relate univariate marginal distributions functions, \( F_1(x_1), F_2(x_2), ..., F_p(x_p) \), to their joint distribution function

\[
C(F_1(x_1), F_2(x_2), ..., F_p(x_p)) = F(x_1, x_2, ..., x_p)
\]

(1)

Each random variable’s univariate marginal distribution may be chosen according to its features. The copula function does not constrain the choice of marginal distribution.

Sklar (1959) (cit in Frees and Valdez (1998)) prove the converse, meaning that any multivariate distribution function, \( F \), can be written in the form of equation (1). He also showed that if each marginal distribution function is continuous, then there is a unique copula representation.

Copula functions have been used by biological science to analyse the joint mortality pattern of groups of individuals. Li (2000) applied this concept to default correlation between companies. Schonbucher and Schubert (2001) use other concept of biological studies – that models heterogeneity via random effects, the frailty model – to study default correlations within an intensity model.
The copula summarizes different types of dependence even when they had been scaled by strictly monotone transformations (invariance property).

The properties of bivariate copula functions, $C(u, v, \rho)$, where $u$ and $v \in (0, 1)^2$ and $\rho$ is a correlation parameter (it can be Pearson correlation coefficient, Spearman’s Rho, Kendall’s Tau or none of these) are as follows:

(i) since $u$ and $v$ are positive numbers, $C(0, v, \rho) = C(u, 0, \rho) = 0$;

(ii) the marginal distribution can be obtained by $C(1, v, \rho) = v$ or $C(u, 1, \rho) = u$

(iii) if $u$ and $v$ are independent variables, $C(u, v, \rho) = u v$

(iv) the upper and lower bound for a copula function is

$$\max (0, u + v - 1) \leq C(u, v) \leq \min (u, v)$$

The generalization of these properties to higher dimensions is straightforward.

The joint distribution function is defined by its marginals and the copula. This means that we may examine the copula function to capture the association among random variables. Once again, we consider only two random variables. Both Spearman’s Rho, $\rho_S$, and Kendall’s Tau, $\tau$, can be defined in terms of the copula function as follows

$$\rho_S = 12 \int \int [C(u, v) - u v] du dv \quad (2)$$

$$\tau = 4 \int \int C(u, v) dC(u, v) - 1 \quad (3)$$
These non parametric correlation measures do not depend on marginal distributions and are not affected by non linear transformations, like Pearson correlation coefficient.

Mendes and Souza (2004) argue that the copula density function splits the joint distribution function on parameters of the margins, $\gamma$, and parameters of the dependence structure, $\delta$. To fit a copula to bivariate data we must maximize the log-likelihood function, $\iota$

$$
\iota = (u, v, \gamma_u, \gamma_v, \delta) = \log \left[ c(F_u(u; \gamma_u), F_v(v; \gamma_v); \delta) \right] + \log f_u(u; \gamma_u) + \log f_v(v; \gamma_v)
$$

(4)

where $c$ is the copula density function and $f$ is the marginal density function\(^3\). Durrleman,Nikeghbali and Roncalli (2000) present some methods to choose the right copula. We can, however, rely on standard measures like AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion).

3. Economic Intuition

Over the last decades, the dependence between financial markets has been widely studied. These studies have essentially empirical concerns related with portfolio diversification and financial integration. Only recently, given important events, like Asian crisis, the burst of new economy’s bubble and fraudulent bankruptcies of Enron and WorldCom, researchers started looking to the dependence structure between companies’ extreme events. BCBS (1999) refers that practitioners have been managing

\(^3\) See Mendes and Souza (2004) for an example of the fitting process. The authors assume that the margins of IBOVESPA and S&P500 follow a $t$-student distribution and fit four copulas: the $t$-student, the BB1, the Gumbel and the Gaussian copula.
and studying this dependence, but there is a lack of theoretical and empirical work on this issue that checks the robustness of the frameworks.

Our motivation is to provide some empirical evidence on the latent factors that drive default correlations. In this section, we present the economic intuition behind this study and the hypothesis we are going to test.

It is evident that the theoretical evolution of the structural approach has been towards the approximation of the features of the real economy. The empirical studies by Jones, Mason and Rosenfeld (1984), Huang and Huang (2003) and Eom, Helwege and Huang (2004) report that these models tend to systematically underestimate observed yield spreads and, given the high dispersion of predicted spreads, are inaccurate. In our opinion, this does not affect the accuracy of the structural model in estimating EDPs. The existing studies on observed yield spread do not consider all the relevant components that affect the yield spreads. As Fisher (1959) argues, observed bond yield spread provides compensation to investor for credit risk and marketability risk. Several authors (see Delianedis and Geske (2001) and Ericsson and Reneby (2005)) refer that default spread is only a small proportion of the observed yield spread. On the other hand, Leland (2002), Patel and Vlamis (2004) and Patel and Pereira (2005) show that EDPs from structural models are able to predict companies’ bankruptcies, in some cases up to two years before the event. In this respect structural approach provides a useful underpinning for estimation of default probabilities for firms that are closer to economic/financial distress.
Besides the differences in the variables that are stochastic and deterministic, we can identify two classes of models: Merton (1974) model, that considers default only at the maturity of the zero coupon bonds; and those models, that consider default at any time during the prediction horizon – first passage models. We must also note that whilst the models work with risk neutral measures, the resulting expected default probability can differ from the real one and is likely to be higher. This happens because the drift of the real process for assets’ value (see Appendix A1) normally, is higher than the risk neutral one. Thus, we expect to observe Expected Default Probability resulting from the first class of models to be lower than the one resulting from the second class of models.

Malkiel and Xu (2000) argue that investors price idiosyncratic risk because they cannot hold a diversifiable portfolio. Similar evidence is found by Goyal and Santa-Clara (2003). Therefore, if investors, for some exogenous reason, hold an undiversifiable portfolio, we should expect that idiosyncratic risk increase in periods of companies’ financial distress.

Giesecke (2004) argues that macro-economic variables and operational and/or financial ties can explain default correlations between companies. Schonbucher (2003) says that default correlation spreads by other channels than business ties and Collin-Dufresne et al (2003) focus on default contagion. Both studies consider that information is not available to all investors and so default correlation appears via information effects, learning effects or updating of beliefs, which means that the default of one company provides information about the default risk of other companies. According to the authors, in the past, events like unexpected or fraudulent defaults leaded to market-wide jumps in credit spreads, which narrow the ability to diversify this risk. In our empirical
analysis, we, therefore, expect to observe common or latent factors that drive default correlations, which are related to overall economic and bond markets variables.

4. Data and Methodology

The data stock prices and financial data on US bankrupt companies used in this study are obtained from the Datastream and Osiris database. The names of US bankrupt companies are collected from Moody’s Reports (2003, 2005). A company is classified as bankrupt if it missed or delayed disbursement of interest or principal or if it entered into liquidation, receivership or administration. Using these data sources, an initial sample of 56 bankrupt companies and 59 bankruptcy events between 1996 and 2004 was compiled. In order to ensure reliability of the results, we excluded thinly traded companies (when there is more than 10 days without any trade) and companies with less than 5 years of financial data. The remaining sample comprises 34 bankrupt companies, a total of 36 bankruptcy events and 282 yearly observations. For risk-free rate we use the yield on 1-year treasury constant maturity (TCM) securities, from 1990 to 2004, reported by US government securities dealers to the Federal Reserve Bank of New York, which is obtained from the Federal Reserve Board of Statistics.

Our empirical methodology consists of three stages: the first stage involves an estimation of companies’ EDPs, using the structural models described in Appendix A1. Prediction-oriented and information-related tests are employed to infer the performance of those models; the second stage involves information-related tests and an estimation of companies’ idiosyncratic risk; the third and last stage involves factor analysis of companies’ default correlation matrix and analyse of the latent factors.
At the first stage, each of the models outlined in Appendix A1 have a set of parameters that we either estimate or assume as given. Table 1 describes the parameters and how they are computed in our analysis. Our calibration approach is not very different from the standard one employed in previous studies except that the focus here is solely on the parameters needed to compute the EDPs.

Ideally, to apply these structural models, we should have companies with simple capital structures (only the equity and zero coupon bonds). However, since this is not possible, we assume that company’s debt and liabilities can be converted on a zero coupon bond with face value equal to its debt and liabilities value and maturity of 1 year.

To compute company’s market value and volatility we adopt an iterative procedure using Ito’s Lemma\(^4\) (based on KMV procedure). As a first estimate of company’s volatility, \(\sigma_V\), we use the volatility of the past 12 months equity’s daily returns, \(\sigma_E\). Using equation (A4) and each trading day of the last year, we compute company’s value, \(V_t\), using the market value of equity of that day. This procedure allows us to obtain daily values for \(V_t\) and \(\sigma_V\), which is used for the next iteration. We repeat this procedure until the values of \(\sigma_V\) from two consecutive iterations converge to a value less than 10E-4. Achieved the convergence, we use the value of \(\sigma_V\) to compute \(V_t\) from equation (A4). We consider \(T\) and \(\tau\) equal to one year given investors’ interest in prediction horizons of one year. The parameter \(\delta\) captures the payments made by the

\[^4\] We solve Ito’s equations

\[
\sigma_V = E_t(V_t, \sigma_V, T-t) / V_t \sigma_E N(d_1)
\]

and

\[
E_t(V_t, \sigma_V, T-t) = \hat{E}_t
\]

where \(E_t(V_t, \sigma_V, T-t)\) is the theoretical value of company’s assets, \(\sigma_E\) the volatility of equity, \(N(.)\) the standard normal distribution function, and \(\hat{E}_t\) denotes the observed market value of equity.
company to its shareholders and bondholders, such as dividends, share repurchases and bond coupons. According to Huang and Huang (2003), 6% can be assumed to be a reasonable estimate for this parameter.

Several studies in the literature report that, bondholders’ recovery rate varies with the type of debt. For example, Altman (1992) (cit in Longstaff and Schwartz (1995)) finds that, during the period 1985-1991, the average recovery rate for a sample of defaulted bond issues was: 0.605 for secured debt, 0.523 for senior debt, 0.307 for senior subordinated debt, 0.28 for cash-pay subordinated debt, 0.195 for non cash-pay subordinated debt. Given this evidence, previous studies (e.g. Longstaff and Schwartz (1995), Leland (2002), Huang and Huang (2003), Eom et al. (2004)) assume an average recovery rate of 51% of debt face value.

In one-factor models (M and ER) the risk-free rate used is the yield on 1-year TCM rates available on the last day of the year. In the LS two-factor model, as mentioned in the Appendix A1, the interest rate is driven by Vasicek process described in equation (A1). Eom et al (2004) apply Vasicek and Nelson-Siegel models to estimate the term structure of the risk free yield curve and get similar results. Under this evidence, we fit Vasicek model to 1-year TCM rates assuming that $\sigma_r = 0.015$ (see Appendix A1 and equation (A2) for details about the estimation). We estimate the parameters $a$ and $\lambda$ using this procedure for each year, from 1990 to 2004, with the daily observations of 1-year TCM of that year. The correlation coefficient is computed with 1-year TCM rates and $V_t$ for each common year.

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5 This value is the weighted average, by the average leverage ratio of all firms for S&P 500, between the observed dividend yield and historical coupon rate (during the period 1973-1998). Huang and Huang (2003) also argue that the use of one payout ratio for firms with different credit ratings is not erroneous given that, probably, firms with lower credit rating may have higher debt’s payouts than the ones with higher credit rating but they also are likely to make less payment to shareholders.
At the second stage, we can estimate idiosyncratic volatilities from the residuals of an asset pricing model. Obviously, the results are sensitive to choice of the asset pricing model and on its ability to capture the significant variables. Since the existing literature has tended to employ the three-factor model\footnote{We thank Kenneth French for making this data available at his web page: \url{http://mba.tuck.dartmouth.edu/pages/faculty/ken.french}.} of Fama and French (1993), we also use the model shown in equation (5) below,

\begin{equation}
R_{i,t} - R_{f,t} = \beta_{m,i} (R_{m,t} - R_{f,t}) + \beta_{smb,i} R_{smb,t} + \beta_{hml,i} R_{hml,t} + \epsilon_{i,t}
\end{equation}

where $R_{i,t}$ is the return on stock $i$ in day $t$, $R_{m,t} - R_{f,t}$ is the market excess return, $R_{smb,t}$ is the return on a portfolio that captures the size effect (is the average return on three small portfolios minus the average return on three big portfolios) and $R_{hml,t}$ is the return on a portfolio that captures the book-to-market equity effect (is the average return on two value portfolios minus the average return on two growth portfolios). We fit this model using past year’s daily stock returns and use the standard deviation of $\epsilon_{i,t}$ as proxy for company’s $i$ idiosyncratic risk.

Finally, at the third stage, we start by fitting copula functions, employing equation (4), to each pair of companies’ EDPs, using the common years. To fit the copula functions, we define a minimum of 5 common years, which implies a smaller sample of companies (23 companies and 25 bankruptcy events). After the estimation of the copula function for each pair of companies’ EDPs (a total of 276 copula functions), we transform them to Kendall’s Tau, using equation (3), as outlined in section 2. A correlation matrix is computed for each structural model. Kendall’s Tau is used in a
Factor Analysis (see Appendix A2) to extract the *common* or latent factors, \textit{fact}_t, that are not directly observed. As explained earlier, we assume that the companies’ EDPs are indicators of the construct that explains the default correlations between companies. Our next objective is to identify the variables that explain the correlations. Recent literature documents several variables that explain default risk and, consequently, default correlations. We consider the variables that seem theoretically robust and empirically measurable. Therefore, as determinants of default correlations we take into account:

a) \textit{Treasury Interest Rates Level}. Several authors (e.g. Longstaff and Schwartz (1995), Leland and Toft (1996)) argue that, theoretically, a higher spot rate decreases the EDPs, given the increase in company value process drift. Since majority of the models consider that the threshold is constant or deterministic, increase in the drift pushes company’s value away from threshold value and decreases default probability. Collin-Dufresne and Goldstein (2001) introduce a third stochastic process (mean-reverting), for company’s capital structure, to solve this problem. Since an increase in the level of treasury interest rates implies a decrease in EDPs, we should also expect to observe a decrease in default correlations. This variable, \( r_{t}^{10} \), is measured as the yield on the 10-year TCM securities at the end of the year. In line with Collin-Dufresne, Goldstein and Martin (2001), we consider the square of this variable, \( (r_{t}^{10})^2 \), to capture potential nonlinear effects due to convexity.

b) \textit{Slope of the Yield Curve}. The impact of this variable on default probabilities and default correlations is controversial. In our opinion, since this variable reflects investors’ expectations about the evolution of the economy, an increase in slope of the yield curve implies a strengthening economy and, therefore, a decrease in
EDPs and default correlations. We define this variable as the difference between the 10-year and 2-year TCM yields, \( r_{t}^{10} - r_{t}^{2} \), at the end of each year.

c) Market Volatility. Since structural models expand Black and Scholes (1973) model, volatility is a critical parameter. The effect of volatility depends on the type of model specification. If it is a first-passage model, an increase in volatility increases probability of default and default correlations, because the probability of company’s value crossing the threshold, at any moment of time, also increases. If it is a European type model, like the one of Merton (1974), the effect is not obvious; it can be positive or negative. We measure market volatility, \( \sigma_{S&P} \), as the standard deviation of past 12 months of S & P daily returns.

d) Equity Premium. Economic intuition suggests that equity premium can be considered a proxy for the overall state of the economy. We may expect that an increase in equity premium reflects optimistic expectations about the future and, therefore, a decrease in defaults and default correlations. We measure equity premium, \( R_{M} - r_{t}^{1m} \), as the difference between the value-weighted return on all NYSE, AMEX and NASDAQ and one-month Treasury bill rate.

e) Default Return Spread. This variable, despite being a systematic factor, captures specific information about bond markets such as unexpected or fraudulent bankruptcy. As defended by Schonbucher (2003), this variable can be interpreted as a learning or information effect variable. An increase in default return spread increases an overall bond market uncertainty, which implies an increase in default correlations, because investors become more sensitive to bad
news. Default return spread, DefSpread, is defined as the difference between Moody’s BAA and AAA long term bonds yields\(^7\), at the end of the year.

Table 2 summarizes the expected signs of the sensitivity of default correlations to these variables. The first four variables capture cyclical default correlation while the last captures the systematic component of default contagion effects.

For each factor, resulting from each model’s correlation matrix, we estimate the following regression

\[
\text{Facts}_{t,1} = \alpha + \beta_1 r_{t1}^{10} + \beta_2 (r_{t1}^{10})^2 + \beta_3 (r_{t1}^{10} - r_t^2) + \beta_4 \sigma_{S&P, t} + \beta_5 (R_{M, t} - r_t^{1m}) + \beta_6 \text{DefSpread}_t
\]  

(6)

5. Empirical Evidence

In this section, we report and discuss the results of the performance of the structural models obtained from prediction-oriented and information-related tests. We investigate the importance of idiosyncratic risk in predicting bankruptcy events. We also extract latent factors for companies’ default correlation and identify the determinants of these factors.

EDPs provided by structural models, as well as any value provided by contingent claims models, are highly dependent on two parameters: volatility and debt ratio. Table

\(^7\) Collin-Dufresne et al. (2001) define this variable as the difference between BBB Index Yield and 10-year Treasury Yield, which can bias the spread since these two classes of securities have different degrees of liquidity.
3-5 present cross-sectional results of a multivariate linear regression model. All regressions are statistically significant at 1 percent level. In table 3, we can see that debt ratio and company’s volatility explain a good percentage of the variability of EDPs (only for LS model, this percentage is below 50 percent). All coefficients of these two variables are statistically significant and have the expected sign. EDPs are more sensitive to debt ratio than to company’s volatility. First passage models are more sensitive to debt ratio than M model.

In table 4 we present the results of idiosyncratic risk instead of company’s volatility. The explanatory power of the regressions increases substantially (approximately 70 percent). These two variables explain about 77 percent of the variability of EDPs provided by M model, which is confirms of the importance of idiosyncratic risk to these variables.

In table 5, besides debt ratio, we use company’s volatility and idiosyncratic risk. Compared to the results in table 4, the explanatory power of the regression does not increase substantially, however, it is clear that idiosyncratic risk is the most important variable. The coefficient of idiosyncratic risk is statistically significant. It is surprising to observe that volatility looses importance and in LS and ER model its coefficient is not statistically different from zero.

Prediction-oriented tests

Prediction-oriented tests provide an in sample accuracy measure, which is classified into error type I and II. Since our sample has only bankrupt firms, we only observe Error type I, that is when the model fails to predict bankruptcy. The models correctly
predict bankruptcy if in the final available year EDP is above 20 percent\textsuperscript{8}. The employed models have different performance. Our results show misclassification of 3 bankruptcy events by M and LS models, which correspond to 8.3 percent of the bankruptcy events. ER model is the best model with only 2 misclassified bankruptcy events, which corresponds to 5.5 percent of the bankruptcy events. From this evidence, we conclude that overall the structural models are able to predict corporate bankruptcy at least one year in advance of the event.

**Information-related tests**

This analysis complements the prediction-oriented tests. There are, however, several limitations of this method: first, its assume a dichotomous decision context; second, both error types are equally important (for a credit risk manager, it is more serious to classify a bankrupt firm as non-bankrupt than a non-bankrupt firm as bankrupt); third, the classification of firms as bankrupt or as non-bankrupt is somewhat subjective because it implies the definition of a cut-off value; and forth, it is not clear which model explains better the variability of companies’ default risk. Moreover, this procedure has the limitation of considering bankruptcy as an event and not as a process\textsuperscript{9}.

We use logistic regressions to compute information-related tests, which allows us to rank the EDPs and other variables according to their explanatory power. Based on Shumway (2001), we assume that the relationship between companies’ default risk and independent variable(s) is represented by a logistic curve that asymptotically

\textsuperscript{8} Moody’s Report (2005) presents default rates term structure for several period of time. Default probability of a Caa-C firm, during the period 1920-2004, at 1-year horizon, was around 15 percent. For the period 1983-2004, at the same horizon, was around 22 percent. Standard & Poor’s transition probability from CCC to default is around 19.8 percent (see Crouhy et al. (2000))

\textsuperscript{9} One way to solve this problem is to use lag values on the logistic regression. We did not use this procedure because it would imply the loss of observations and because it is a source of bias fixing a number of lagged values to all companies.
approaches one (zero) as covariates tend to positive (negative) infinity. This relationship is written as follows\textsuperscript{10}

\[
P_{t-1}(Y_{it} = 1) = \frac{1}{1 + \exp(- (\alpha + \beta X_{i,t-1}))}
\]

where \(X_{i,t-1}\) is the vector of time varying covariates, known at the end of previous year, \(\alpha\) denotes the constant. \(Y_{it}\) is the dependent variable and, in a particular year, equals one when a company goes bankrupt and zero otherwise. Each year that a company is alive corresponds to an observation in the estimation equation.

Table 6 reports the results of logistic regressions. Column 1 to 8 displays univariate regressions with the EDPs provided by the structural models\textsuperscript{11} and with idiosyncratic risk, debt ratio, market-to-book assets ratio (MB), book–to-market equity ratio (BM) and volatility. According to Vassalou and Xing (2004), default risk is explained BM ratio, while MB ratio is introduced as a proxy for companies’ growth opportunities. We use Nagelkerke R\(^2\) as an explanatory power indicator.

Models’ EDPs are statistically significant and have the expected sign. M model has the highest explanatory power (around 40 percent), which is in contrast to the results of prediction-oriented tests reported earlier. ER model also shows good performance and is very close to M model. LS model has the lowest performance. The performance of these models is better than reported by Campbell et al. (2004) at the one-month

\textsuperscript{10}This non-linear relationship can be rewritten as a linear one

\[
\ln[P_{t-1}/(1 - P_{t-1})] = \alpha + \beta X_{i,t-1}
\]

where the dependent variable represents the log of the odds.

\textsuperscript{11}Based on Patel and Pereira (2005), we also perform logistic regressions with model-scores. We do not show these results because they are very similar to the ones reported.
horizon. Considering that for some of the failed firms we have lagged financial and accounting data that sometimes goes up to two years, these results are very good.

It is worth pointing out the performance of idiosyncratic risk and debt ratio variables. Both variables have the expected sign and are significant at a 1 percent confidence level. Default risk does not observed to be not sensitive to volatility, and the explanatory power of this coefficient is almost zero. These results show that idiosyncratic risk is an important variable. Comparing the explanatory power of idiosyncratic risk (column 4) and volatility (column 8), it is evident that the former variable predicts bankruptcy events better than the latter variable. This implies that investors are aware of companies’ specific deteriorating circumstances and anticipate bankruptcy, that is why the coefficient of idiosyncratic risk is significant. The coefficient of debt ratio, is significant as expected. It is worth highlighting the explanatory power of ER and LS model, relatively to debt ratio, which, given the complexity of these models, is somewhat intriguing.

In contrast to the results reported by Vassalou and Xing (2004), the explanatory power of BM ratio is almost zero, and this variable is not significant. MB ratio has the expected sign and is significant at 5 percent confidence level, but its explanatory power is very low (see column 6 and 7).

In column 9 and 10, we estimate a stepwise logistic regression with M and ER model, respectively, and idiosyncratic risk and debt ratio. Idiosyncratic risk is always dominated by other variables and M and ER models are statistically more significant than debt ratio. In both regressions, the explanatory power increases substantially.
Finally, according to –2LogL statistic, that has a $\chi^2$ distribution with $n-q$ degrees of freedom, where $q$ is the number of parameters in the model, we cannot reject the null hypothesis of all logistic regressions, which suggests that model fits the data.

*Distribution of EDPs*

Table 7 reports EDPs of each structural model summary statistics, for all years and for the previous 6 years of the analysed period (it is not feasible to present all the results over the period 1990-2004). We consider that the year of the bankruptcy event for the failed companies is the year $n$. The first important observation in Table 7 (and displayed in Figure 1) is that average EDP from M model is lower than the one of LS and ER models. This happens because, as mentioned earlier, the former model treats debt as a European option and the latter ones are a kind of Barrier option. Focusing on the behaviour of EDPs, we observe a gradual increase of the EDPs two years before the event and then a steep increase in the year before the event. The second important observation is that the standard deviation of EDPs provided by M model is around half and two fifth of the ER and LS models, respectively. This means that M model is more accurate than the other two models (see figure 2-4). Within the first passage models, we observe a distinct clustering of these models, but LS model is the least accurate and has more extreme values (see figure 3 and 4).

Overall, the results suggest that the first passage models do not add value to M model. Surprisingly, the two-factor model (LS model) is has the worst performance, suggesting that the effort to capture the real world in this model, as far as EDPs are concerned, is not justified.
Factor Analysis of Correlation Matrix

Next we present results of the joint variability of companies default risk, that is, of the companies default correlation matrix, based on Factor-analytical tests (see Appendix A2 for details). We compute a correlation matrix per model and fit copula functions\textsuperscript{12} by maximizing the log-likelihood function as explained in equation (4) to each pair of EDPs. Given the restrictions outlined in section 4, the sample is consists of 23 companies, which is 25 bankruptcy events and 276 copula functions. The results show that all fitted copula functions belong to the normal family. Second, we construct companies’ correlation matrix with Kendall’s Tau, which is estimated using equation (3). It is this correlation matrix that is used to in Factor Analysis to estimate the factors responsible for the correlations.

We employ principal components method to extract the factors from the correlation matrix. We retain the factors that have an eigenvalue greater than one. Table 8 reports the results of factor analysis for each model. We extract 5 and 6 factors, respectively, for M and LS and ER model. The RMSR and the non-redundant residuals for the residual matrix are small in all models, implying a good factor solution. The five factors of M model and the six factors of LS and ER models, referred to as common or latent factors, explain a high percentage of the observed variance (79.5 percent and 86.5 percent and 82.8 percent, respectively), which encourages us to find the determinants of default correlation. In contrast to Zhou (2001), the results suggest that only a small percentage (21.5, 13.5 and 17.2 percent for M, LS and ER models, respectively) of observed variance or default correlation is explained by non-systematic factors. An

\textsuperscript{12} Several copula families are fit, such as the normal and extreme values families.
orthogonal rotation (Varimax rotation) is performed to achieve a simpler factor structure. We use the rotated component matrix to estimate time series values for each model’s factors.

Since it is more difficult to interpret and analyse the determinants of default correlations with so many factors, we use principal component analysis to reduce the factors. We use the rule of eigenvalue greater than one to retain the new factors. The initial five common factors of M model and six common factors of LS and ER models are reduced to two common factors, for each model. Two common factors explain around 90 percent of the total variability of its initial common factors (Table 9).

Table 10 presents the determinants of default correlation factors. We estimate equation (6) using a stepwise procedure\textsuperscript{13} (because we identified a multicollinearity problem). As expected, given the assumptions of the stepwise procedure, all the variables and all regressions are statistically significant, at 5 percent confidence level. Overall, the regressors’ explanatory power is very high, around 55 percent, with a maximum of 71 percent. Further, the signs of the estimated coefficients generally as expected (see Table 2). Market volatility and equity premium explain 56 percent of the variability of the common factor 1 in M model. This factor can be interpreted as a capital market factor. Equity premium has the expected sign and market volatility has a negative effect on default correlation. Volatility does not explain the variability of default correlation in the first passage models.

\textsuperscript{13} Several studies (e.g. Collin-Dufresne et al. (2001) argue that default probabilities can be explained by nonlinear, cross term and lagged values of regressors (such as squared and cubic slope of the yield curve or \((r_t^{10} - r_t^2)\sigma_{S&P, t}\}). Nevertheless, none of these terms seems to explain default correlations and that is why we restrict this analysis to the variables in equation (6).
The slope of the yield curve explains the variability of the common factor 2 in M
model, but the sign of the estimated coefficient sign is not as expected. One possible
explanation for this is that an increase at the slope of the yield curve makes it more
difficult for distress firms to renegotiate the debt and increases default risk and default
correlations.

We observe some similarity in the variables that explain the common factors in LS and
ER models. Both common factors 1 are explained by the slope of the yield curve,
although this variable has different signs in the regressions. Only the common factor 1
in ER model has the expected sign. Both common factors 2 are explained by treasury
interest rates level and market equity premium that allows us to interpret them as a
return pushed factor. Consistent with Longstaff and Schwartz (1995) and Collin-
Dufresne et al. (2001), we find that an increase in the risk free rate lowers EDPs and
default correlations. The estimated coefficients of equity premium have about the same
magnitude in all models.

The default return spread, proxy for default contagion, is not significant in any of the
regressions, suggesting that either it does not explain default correlations, as argued by
Schonbucher (2003) and Collin-Dufresne et al. (2003), or that this variable is not the a
good proxy. We should point out that, given the nature of this effect, it is probably
better to capture this effect by a non-systematic variable or a variable that considers
companies’ business and financial ties. Convexity is not significant in any of the
regressions, which is consistent to the findings of Collin-Dufresne et al. (2001).
Ljung-Box test indicates that standardized residuals from the regressions are not autocorrelated (the average serial correlation of standardized residuals is 0.02) and the average Durbin-Watson statistic is 1.81.

In summary, default correlations are driven essentially by common factors that explain on average around 83 percent of total variance. Only a small percentage of default correlation is due to non-systematic factors. These results are consistent with economic intuition and empirical evidence (see Vassalou and Xing (2004)) according to which, during recession periods, default risk increases and a cluster of bankruptcy events is observed. What drives default correlations are: first, capital market equity premium and treasury interest rates level, which reflects the overall state of the economy. This is consistent with the theoretical intuition of Hull et al. (2005) when they argue that the systematic variable that drives default correlations is capital market wiener process. Second, the slope of the yield curve reflects investors’ expectations about the evolution of the economy. As far as default correlations are driven basically by systematic factors, portfolio diversification should reduce default risk.

6. Conclusion

In this study we analyse the determinants of default correlations for a sample of US bankrupt companies. We apply a set of structural models (Merton (1974), Longstaff and Schwartz (1995) and Ericsson and Reneby (1998)) to estimate companies’ EDPs. Given that we observe a sharp increase in EDPs up to two years in advance of default event, these models provide timely and accurate estimates of companies default risk.
Another novel finding is the importance of idiosyncratic risk (and not of total volatility) in predicting default events. This suggests that company specific signals provide useful information to investors about the deterioration in company’s economic and financial conditions prior to bankruptcy.

We compute companies’ default correlation matrix using a copula function and employ Factor Analysis technique to extract factors that explain companies’ default correlations. The results of prediction-oriented tests suggest that ER model is the best model as it misclassifies 5.5 percent of bankruptcy events. The results of information-related tests suggest that M and LS model have a similar performance. Variables such as market-to-book asset ratio and book-to-market equity ratio, which other studies have found to be significant, have poor explanatory power in our regression analysis. We observe that common factors explain around 83 percent of the variability of default correlations. This evidence supports the belief that common factors are explained by the overall state of the economy and by the expectations of its evolution.
References


Appendix A1: Structural Models

In this section, we present a brief summary of the models by Merton (1974), Longstaff and Schwartz (1995) and Ericsson and Reneby (1998). Since our main concern here is with the empirical performance of these models, we do not discuss in detail the theoretical properties of the models. Throughout this section, we assume that uncertainty in the economy is modelled by a filtered probability space \((\Omega, \mathcal{G}, P)\), where \(\Omega\) represents the set of possible states of nature, \(\mathcal{G}_t\) is the information available to investors over time \(t\) and \(P\) is the probability measure. All models assume a perfect and arbitrage-free capital market, where risky and default-free bonds and companies’ equity are traded. The risk-free numeraire (or money market account) value, at time \(t\), \(A_t\), follows the process

\[
A_t = \exp(\int_0^t r_s ds)
\]

where \(r\) denotes the short-term risk-free interest rate, which can be deterministic or modelled by a stochastic process. When modelled as a stochastic process, the dynamics of \(r\) is driven by a Vasicek-model

\[
dr_t = a(\lambda - r_t)dt + \sigma_r dW^r \tag{A1}
\]

where \(a\) is the short term interest rate mean reversion speed, \(\lambda\) and \(\sigma_r\) are its mean reversion level and standard deviation, respectively. The variable \(dW^r\) is a Wiener process. In this economy, the investors are assumed to be risk-neutral, which means that the probability measure, \(P\), is a martingale with respect to \(A_t\). The value of a riskless discount bond that matures at \(T\) is (Vasicek, 1977):

\[
D(r, T) = \exp(A(T) - B(T)r) \tag{A2}
\]

\[
A(T) = \left(\frac{\sigma_r^2}{2a} - \lambda\right)T + \left(\frac{\sigma_r^2}{a^2} - \frac{a\lambda}{a^2}\right)\exp(-aT) - 1 - \left(\frac{\sigma_r^2}{4a^2}\right)\exp(-2aT) - 1
\]
Under the risk neutral probability space, the value of the company’s assets, $V$, follows a geometric brownian motion ($\mathcal{G}_t$ – adapted diffusion process) given by

$$dV_t = (r_t - \delta)V_t dt + \sigma_V V_t dW^V$$

(A3)

where $\delta$ denotes company’s assets payout ratio and $\sigma_V$ company’s assets volatility. The variable $dW^V$ is a Wiener process under the risk-neutral probability measure. $\rho$ is the instantaneous correlation coefficient between $dW^f$ and $dW^v$.

The dynamics of company’s assets value, under the real probability space, is given by

$$dV_t = (\mu - \delta)V_t dt + \sigma_V V_t dW^{PV}$$

where $\mu$ denotes company’s assets expected total return and $dW^{PV}$ is a Wiener process under the real probability measure. For the dynamic process described by equation (A3), and the given assumptions, the standard hedging framework leads to the following partial differential equation

$$\frac{1}{2} \sigma_V^2 V^2 \frac{\partial^2 F}{\partial V^2} + (r - \delta)V \frac{\partial F}{\partial V} - rF + \frac{\partial F}{\partial t} + P = 0$$

where $F$ is the price of any derivative security, whose value is a function of the value of the firm, $V$, and time, and $P$ represents the payments received by this security. The two-factor models by Longstaff and Schwartz (1995) assume that $F$ is a function of the value of the firm, $V$, time and interest rates.

The standard hedging framework leads to the following partial differential equation

$$\frac{1}{2} \sigma_V^2 V^2 \frac{\partial^2 F}{\partial V^2} + \rho \sigma_V \sigma_r V \frac{\partial F}{\partial V \partial r} + \frac{1}{2} \sigma_r^2 \frac{\partial^2 F}{\partial r^2} + (r - \delta)V \frac{\partial F}{\partial V} + (a\lambda - ar) \frac{\partial F}{\partial r} + \frac{\partial F}{\partial t} - rF + P = 0$$
Given our focus on the empirical performance of structural models in predicting corporate failure, we only outline the equations relevant for the expected default probability in each model. We refer the reader to the original papers for the full description the models.

**Merton (M) Model:** Merton (1974) model is an extension of the Black and Scholes (1973) option pricing model to value corporate securities. The company’s assets value, which corresponds to the sum of the equity and debt values, is driven by the process described by equation (A3) and is assumed to be independent of company’s capital structure. Under these assumptions, equity value, $E_t$, is defined by a call option on the assets of the firm, with maturity $T$ and exercise price $F$:

$$E_t = V_t N(d_1) - e^{r(T-t)} F N(d_2)$$  \hspace{1cm} (A4)

where

$$d_1 = \frac{\ln \left( \frac{V_t}{F} \right) + \left( r + 0.5\sigma^2 \right)(T-t)}{\sigma \sqrt{T-t}}, \quad d_2 = d_1 - \sigma \sqrt{T-t}$$

and $N(.)$ represents the standard normal distribution function. Debt’s value, at time $t$, is equal to:

$$D_t = V_t - E_t$$

If at maturity, company’s assets value, $V_T$, is higher than the face value of its debt, $F$, the firm does not default, bondholders receive $F$ and shareholders $V_T - F$. However, if $V_T < F$, the firm defaults and there is a transfer of company’s ownership from shareholders to bondholders. Firm only defaults at time $T$, and $N(-d_2)$ represents the risk-neutral probability of default.
**Longstaff and Schwartz (LS) Model:** Longstaff and Schwartz (1995) develop a two factor model to value risky debt, extending the one-factor model of Black and Cox (1976) in two ways: (i) incorporating both default risk and interest rate risk; (ii) allowing for deviations from strict absolute priority. An important feature of this model is that firms with similar default risk can have different credit spreads if their assets have different correlations with changes in interest rates. Their assumptions are not very different from the ones used by Black-Scholes, Merton (1974) and Black and Cox (1976), except for the fact that short term risk free interest rate follows the dynamics described by equation (A1) (and the riskless discount bond can be priced using equation (A2)) and that there are bankruptcy costs, $\alpha$. The default boundary, $K$, is constant and exogenously specified, which is consistent with the assumption of a stationary capital structure. Setting $X$ equal to the ratio $V/K$, the price of a risky discount bond that matures at $T$ is

$$D(X, r, T) = D(r, T) - \alpha D(r, T)Q(X, r, T) \quad (A5)$$

where

$$Q(X, r, T, n) = \sum_{i=1}^{n} q_i \quad (A6)$$

$$q_i = N(a_i)$$

$$q_i = N(a_i) - \sum_{i=1}^{n} q_i N(b_{ij}) \quad i = 2, 3, ..., n,$$

$$a_i = -\ln X - M(jT/n, T) \sqrt{S(iT/n)}$$

$$b_{ij} = \frac{M(jT/n, T) - M(iT/n, T)}{\sqrt{S(iT/n) - S(jT/n)}}$$

and where

$$M(t, T) = \left( \frac{a\lambda - \rho \sigma_x \sigma_r}{a} - \frac{\sigma_x^2}{2a^2} \right) t + \left( \frac{\rho \sigma_x \sigma_r}{a^2} + \frac{\sigma_r^2}{2a^3} \right) \exp(-aT)(\exp(at) - 1)$$

$$+ \left( \frac{r}{a} - \frac{a\lambda}{a^2} + \frac{\sigma_x^2}{a^3} \right)(1 - \exp(-at)) - \left( \frac{\sigma_r^2}{2a^3} \right) \exp(-aT)(1 - \exp(-at))$$
The term $Q(X, r, T)$ is the limit of $Q(X, r, T, n)$ when $n \to \infty$ (the authors argue that the convergence between these terms is rapid and that when $n = 200$, the differences between the results of the terms are virtually indistinguishable).

The first term in equation (A5) represents the value of a riskless bond. The second term represents a discount factor for the default of the bond. The factor can be decomposed into two components: $\alpha D(r, T)$ is the present value of the writedown on the bond if default occurs; $Q(X, r, T)$ is the probability, under the risk neutral measure, that a default occurs (this probability can differ from the real one).

**Ericsson and Reneby (ER) Model:** Ericsson and Reneby (1998) demonstrate that corporate securities can be valued as portfolios of three basic claims: a down-and-out option that expires worthless if the underlying variable reaches a pre-specified lower boundary, prior to the expiration date; a down-and-out binary option that yields a unit payoff at the expiration date if the underlying asset exceeds the exercise price; and unit down-and-in option that pays off one unit the first time the underlying variable reaches a lower boundary. This formulation allows to value finite maturity coupon debt with bankruptcy costs, corporate taxes and deviations from the absolute priority rule. The default is triggered if company’s value falls below a constant $K$ (the reorganization barrier), at any time prior to maturity of the firm, or if, at debt’s maturity, company’s value is less than some constant $F$, which normally is debt’s face value. The time of default is denoted $\tau$. The price of a unit down-and-in option, that matures at $T$ and pays one monetary unit if bankruptcy happens before $T$ and zero otherwise, is
\[ G^K \{ V_t, t \mid \tau \leq T \} = G^K \{ V_t \mid \tau \leq \infty \}(1 - Q^{G} \{ \tau > T, V_t > K \}) \]  \hspace{1cm} (A7)

where

\[ G^K \{ V_t \mid \tau \leq \infty \} = (V_t / K)^{\theta} \]

\[ Q^{G} \{ \tau > T, V_t > K \} = \int \int_{x > K} \left( \frac{\partial F}{\partial x} \right) dx d\mu \]

\[ d^G_t(s) = \frac{\ln x}{\sigma \sqrt{t}} + \mu^G \sqrt{t} \]

\[ \mu^g = \frac{r - \delta - 0.5\sigma^2}{\sigma} \hspace{1cm} \mu^G = \mu^g - \theta \sigma \hspace{1cm} \theta = \frac{\sqrt{(\mu^h)^2 + 2r + \mu^h}}{\sigma} \]

Equation (A10) represents the expected default probability.
Appendix A2: Factor Analysis

A full understanding of Factor Analysis can be done in Sharma (1996). Factor Analysis use the correlation matrix to: identify the smallest number of common factors (via factor rotation) that best explain the correlation among the variables; and provide an interpretation for these common factors.

This technique assumes that the total variance of a variable can be divided into the variance explained by the common factor and the one explained by a specific factor. A factor model that contains \( m \) factors can be represented as

\[
\begin{align*}
x_1 &= \lambda_{11} \xi_1 + \lambda_{12} \xi_2 + \ldots + \lambda_{1m} \xi_m + \epsilon_1 \\
x_2 &= \lambda_{21} \xi_1 + \lambda_{22} \xi_2 + \ldots + \lambda_{2m} \xi_m + \epsilon_2 \\
&\vdots \\
x_p &= \lambda_{p1} \xi_1 + \lambda_{p2} \xi_2 + \ldots + \lambda_{pm} \xi_m + \epsilon_p
\end{align*}
\]

where \( x_1, x_2, \ldots, x_p \) are variables of the \( m \) factors, \( \lambda_{pm} \) is the pattern loading of the \( p^{th} \) variable on the \( m^{th} \) factor and \( \epsilon_p \) is the specific factor for the \( p^{th} \) variable. The previous construct can be represented as

\[
x = \Lambda \xi + \epsilon \tag{A8}
\]

\( x \) is a \( p \times 1 \) vector of variables, \( \Lambda \) is a \( p \times m \) matrix of factor pattern loadings, \( \xi \) is a \( m \times 1 \) vector of latent factors and \( \epsilon \) is a \( p \times 1 \) vector of specific factors. Equation (A8) is the factor analysis equation. The assumptions are: the common factors are not correlated with the specific factors and the means and variances of variables and factors are zero and one, respectively. Variables’ correlation matrix, \( R \), is

\[
R = \Lambda \Phi \Lambda' + \Psi \tag{A9}
\]
Λ is the pattern loading matrix, Φ is factors’ correlation matrix and Ψ is a diagonal matrix of the specific variances. R - Ψ gives us the variance explained by the common factors. The off-diagonals of R are the correlation among variables. Factor analysis estimate parameter matrices given the correlation matrix. The correlation between the variables and the factors is given by

$$\Lambda = \Lambda \Phi$$

If the m factors are (not) correlated, the factor model is referred to as an oblique (orthogonal) model. In an orthogonal model, it is assumed that Φ = I. Orthogonal rotation technique implies the identification of a matrix, C, such that the new loading matrix is given by $\Lambda^* = \Lambda C$ and $R = \Lambda^* \Lambda^{*\prime}$.

Varimax rotation technique estimate matrix C such that each factor will by a set of different variables. This is achieved by maximizing the variance of the squared loading pattern across variables, subject to the constraint that the communality of each variable is unchanged. C is obtained maximizing the following equation, subject to the constraint that the common variance of each variable remains the same.

$$pV = \sum_{j=1}^{m} \sum_{i=1}^{n} \lambda_{ij}^4 - \frac{\sum_{j=1}^{m} (\sum_{i=1}^{p} \lambda_{ij}^2)^2}{p}$$

where $V$ is the variance explained by the common factors.
### Table 1 – Estimation of Models Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model</th>
<th>Estimated as</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Firms’ Specific Parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_t$ Company’s Value</td>
<td>All</td>
<td>Ito’s lemma</td>
</tr>
<tr>
<td>$\sigma_V$ Company’s Volatility</td>
<td>All</td>
<td>Ito’s lemma</td>
</tr>
<tr>
<td>$F$ Debt’s Face Value</td>
<td>M</td>
<td>book value of total liabilities</td>
</tr>
<tr>
<td>$T$ Years to Maturity</td>
<td>All</td>
<td>assumed 1 year</td>
</tr>
<tr>
<td>$\delta$ Payout Ratio</td>
<td>ER</td>
<td>assumed at 6 percent</td>
</tr>
<tr>
<td>$\tau$ Prediction Horizon</td>
<td>ER</td>
<td>assumed 1 year</td>
</tr>
<tr>
<td>$\alpha$ Bankruptcy Costs</td>
<td>LS</td>
<td>Assumed at 49 percent</td>
</tr>
<tr>
<td>$K$ Threshold Value / Distress Barrier</td>
<td>LS; ER</td>
<td>debt’s face value</td>
</tr>
<tr>
<td><strong>Interest Rate Parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r$ Interest Rate</td>
<td>All</td>
<td>1-year TCM</td>
</tr>
<tr>
<td>$a$ Mean Reversion Speed</td>
<td>LS</td>
<td>Vasicek risk free yield curve</td>
</tr>
<tr>
<td>$\lambda$ Mean Reversion Level</td>
<td>LS</td>
<td>Vasicek risk free yield curve</td>
</tr>
<tr>
<td>$\sigma_r$ Short Rate Standard Deviation</td>
<td>LS</td>
<td>Assumed at 1.5 percent</td>
</tr>
<tr>
<td>$\rho$ Correlation Coefficient between $r$ and $V_t$</td>
<td>LS</td>
<td>computed</td>
</tr>
</tbody>
</table>
Table 2 – Expected Signs of the Sensibility of Default Correlations to Explanatory Variables

\[ \text{Fact}_{i,t} = \alpha + \beta_1 r_{i,10} + \beta_2 (r_{i,10})^2 + \beta_3 (r_{i,10} - r_{i,2}) + \beta_4 \sigma_{S&P,t} + \beta_5 (R_{M,t} - r_{1m}) + \beta_6 \text{DefSpread}_t \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Predicted Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_{i,10} )</td>
<td>-</td>
</tr>
<tr>
<td>( r_{i,10} - r_{i,2} )</td>
<td>-</td>
</tr>
<tr>
<td>( \sigma_{S&amp;P} )</td>
<td>±</td>
</tr>
<tr>
<td>( R_{M} - r_f )</td>
<td>-</td>
</tr>
<tr>
<td>( \text{DefSpread} )</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 3 – Cross-section Analysis: Multiple Regressions (Volatility)

\[ \text{EDP} = a + b \text{Debt Ratio} + c \text{Volatility} + \varepsilon_1 \]

<table>
<thead>
<tr>
<th>Model</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>F</th>
<th>Adj R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merton</td>
<td>-0.344</td>
<td>-10.9</td>
<td>0.704</td>
<td>16.3</td>
<td>0.26</td>
</tr>
<tr>
<td>LS</td>
<td>-0.706</td>
<td>-8.3</td>
<td>1.601</td>
<td>13.8</td>
<td>0.344</td>
</tr>
<tr>
<td>ER</td>
<td>-0.632</td>
<td>-9.9</td>
<td>1.474</td>
<td>16.9</td>
<td>0.333</td>
</tr>
</tbody>
</table>

** Confidence level at 1%.

Table 4 – Cross-section Analysis: Multiple Regressions (Idios. Risk)

\[ \text{EDP} = a + b \text{Debt Ratio} + c \text{Idiosyncratic Risk} + \varepsilon_1 \]

<table>
<thead>
<tr>
<th>Model</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>F</th>
<th>Adj R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merton</td>
<td>-0.368</td>
<td>-18.1</td>
<td>0.263</td>
<td>8.7</td>
<td>0.543</td>
</tr>
<tr>
<td>LS</td>
<td>-0.845</td>
<td>-13.1</td>
<td>0.871</td>
<td>9.1</td>
<td>1.001</td>
</tr>
<tr>
<td>ER</td>
<td>-0.716</td>
<td>-15.1</td>
<td>0.837</td>
<td>11.9</td>
<td>0.834</td>
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</tbody>
</table>

** Confidence level at 1%.
Table 5 – Cross-section Analysis: Multiple Regressions (Volatility / Idios. Risk)

EDP = a + b Debt Ratio + c Volatility + d Idiosyncratic Risk + ε_i

<table>
<thead>
<tr>
<th>Model</th>
<th>a</th>
<th>t-stat</th>
<th>b</th>
<th>t-stat</th>
<th>c</th>
<th>t-stat</th>
<th>d</th>
<th>t-stat</th>
<th>F</th>
<th>Adj R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merton</td>
<td>-0.415</td>
<td>-19.9</td>
<td>0.361</td>
<td>10.9</td>
<td>0.103</td>
<td>5.9</td>
<td>0.476</td>
<td>19.6</td>
<td>365.1**</td>
<td>0.795</td>
</tr>
<tr>
<td>LS</td>
<td>-0.854</td>
<td>-12.2</td>
<td>0.889</td>
<td>8</td>
<td>0.02</td>
<td>0.3</td>
<td>0.988</td>
<td>12.1</td>
<td>145.4**</td>
<td>0.607</td>
</tr>
<tr>
<td>ER</td>
<td>-0.749</td>
<td>-14.7</td>
<td>0.908</td>
<td>11.2</td>
<td>0.074</td>
<td>1.7</td>
<td>0.533</td>
<td>13.2</td>
<td>212.1**</td>
<td>0.693</td>
</tr>
</tbody>
</table>

** Confidence level at 1%.

Table 6 – Logistic Regressions

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>5.356</td>
<td>(0.000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.856</td>
<td>(0.000)</td>
<td></td>
<td></td>
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<tr>
<td>LS</td>
<td>1.839</td>
<td>(0.000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.542</td>
<td>(0.011)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ER</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Idiosyncratic Risk</td>
<td>3.353</td>
<td>(0.000)</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>Debt Ratio</td>
<td>9.326</td>
<td>(0.000)</td>
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<td></td>
<td>4.439</td>
<td>(0.008)</td>
<td>4.557</td>
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<tr>
<td>MB Assets</td>
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<td></td>
<td>-1.160</td>
<td>(0.012)</td>
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<tr>
<td>BM Equity</td>
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<td>-0.010</td>
<td>(0.291)</td>
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<tr>
<td>Volatility</td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td>-0.411</td>
<td>(0.618)</td>
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<tr>
<td>Constant</td>
<td>-3.475</td>
<td>(0.000)</td>
<td>-3.025</td>
<td>(0.000)</td>
<td>-3.563</td>
<td>(0.000)</td>
<td>-4.642</td>
<td>(0.000)</td>
<td>-9.004</td>
<td>(0.000)</td>
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<td></td>
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<td>(0.240)</td>
<td>-1.922</td>
<td>(0.000)</td>
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<td></td>
<td>-1.800</td>
<td>(0.000)</td>
<td>-6.374</td>
<td>(0.000)</td>
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<tr>
<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td>-6.263</td>
<td>(0.000)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>282</td>
<td>282</td>
<td>282</td>
<td>282</td>
<td>282</td>
<td>282</td>
<td>282</td>
<td>282</td>
<td>282</td>
<td>282</td>
</tr>
<tr>
<td>-2 Log L</td>
<td>148.3</td>
<td>166.5</td>
<td>154.8</td>
<td>170.9</td>
<td>154.9</td>
<td>204.8</td>
<td>214.4</td>
<td>214.9</td>
<td>137.4</td>
<td>147.8</td>
</tr>
<tr>
<td>Nagelkerke R²</td>
<td>0.397</td>
<td>0.298</td>
<td>0.362</td>
<td>0.273</td>
<td>0.361</td>
<td>0.069</td>
<td>0.007</td>
<td>0.003</td>
<td>0.452</td>
<td>0.399</td>
</tr>
</tbody>
</table>

p-values in parentheses. The –2LogL statistic has a χ^2 distribution with n-q degrees of freedom, where q is the number of parameters in the model. To all logistic regressions, we cannot reject the null hypothesis, which means that model fits the data (the χ^2 statistics are corrected according to Shumway (2001) suggestions).
### Table 7 – Structural Models EDPs Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Total Sample</th>
<th>n-1</th>
<th>n-2</th>
<th>n-3</th>
<th>n-4</th>
<th>n-5</th>
<th>n-6</th>
<th>All years</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>M</strong></td>
<td>Mean</td>
<td>0.57</td>
<td>0.35</td>
<td>0.24</td>
<td>0.19</td>
<td>0.09</td>
<td>0.09</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation</td>
<td>0.27</td>
<td>0.27</td>
<td>0.26</td>
<td>0.24</td>
<td>0.16</td>
<td>0.17</td>
<td>0.25</td>
</tr>
<tr>
<td><strong>LS</strong></td>
<td>Mean</td>
<td>1.23</td>
<td>0.80</td>
<td>0.55</td>
<td>0.39</td>
<td>0.25</td>
<td>0.22</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation</td>
<td>0.66</td>
<td>0.74</td>
<td>0.61</td>
<td>0.45</td>
<td>0.61</td>
<td>0.49</td>
<td>0.60</td>
</tr>
<tr>
<td><strong>ER</strong></td>
<td>Mean</td>
<td>1.11</td>
<td>0.74</td>
<td>0.53</td>
<td>0.47</td>
<td>0.22</td>
<td>0.21</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>Standard Deviation</td>
<td>0.43</td>
<td>0.48</td>
<td>0.50</td>
<td>0.58</td>
<td>0.37</td>
<td>0.37</td>
<td>0.50</td>
</tr>
</tbody>
</table>

### Table 8 – Factor Analysis Factors

<table>
<thead>
<tr>
<th>Factors</th>
<th>Merton Model</th>
<th></th>
<th></th>
<th>LS Model</th>
<th></th>
<th></th>
<th>ER Model</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Eigenvalue</td>
<td>Cumulative %</td>
<td>Eigenvalue</td>
<td>Cumulative %</td>
<td>Eigenvalue</td>
<td>Cumulative %</td>
<td>Eigenvalue</td>
<td>Cumulative %</td>
<td>Eigenvalue</td>
</tr>
<tr>
<td>1</td>
<td>7.6</td>
<td>32.8</td>
<td>7.1</td>
<td>31.0</td>
<td>6.9</td>
<td>30.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5.8</td>
<td>57.9</td>
<td>4.9</td>
<td>52.3</td>
<td>5.3</td>
<td>53.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.0</td>
<td>66.5</td>
<td>2.8</td>
<td>64.5</td>
<td>2.4</td>
<td>63.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.9</td>
<td>74.8</td>
<td>2.4</td>
<td>75.2</td>
<td>1.9</td>
<td>72.0</td>
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<td></td>
</tr>
<tr>
<td>5</td>
<td>1.1</td>
<td>79.5</td>
<td>1.5</td>
<td>81.8</td>
<td>1.5</td>
<td>78.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.1</td>
<td>86.5</td>
<td>1.0</td>
<td>82.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Root Mean Square Residual (RMSR) = $\sqrt{\frac{\sum_{i=1}^{n} \sum_{j=1}^{p} r_{ij}^2 / (p-1)/2}{p}}$, where $p$ is the number of companies and $res$ gives the amount of correlation that is not explained by the retained factors. Nonredundant residuals are computed as a percentage of the number of nonredundant residuals with absolute values greater than 0.10.

### Table 9 – Principal Components

<table>
<thead>
<tr>
<th>PCA Factors</th>
<th>Merton Model</th>
<th></th>
<th></th>
<th>LS Model</th>
<th></th>
<th></th>
<th>ER Model</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Eigenvalue</td>
<td>% of variance</td>
<td>Eigenvalue</td>
<td>% of variance</td>
<td>Eigenvalue</td>
<td>% of variance</td>
<td>Eigenvalue</td>
<td>% of variance</td>
<td>Eigenvalue</td>
</tr>
<tr>
<td>1</td>
<td>2.7</td>
<td>55.0</td>
<td>2.9</td>
<td>49.0</td>
<td>3.7</td>
<td>62.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.7</td>
<td>34.3</td>
<td>2.1</td>
<td>39.4</td>
<td>1.7</td>
<td>29.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Cum. % explained by PCA: 89.3, 88.4, 91.2
Table 10 – Determinants of Default Correlation Factors

For each model $s$ and each default correlation factor, 1 or 2, we estimate the following regression: $\text{Fact}_{s,t} = \alpha + \beta_1 r_{t}^{10} + \beta_2 (r_{t}^{10})^2 + \beta_3 (r_{t}^{10} - r_{t}^2) + \beta_4 \sigma_{S&P, t} + \beta_5 (R_{M, t} - r_{t}^{1m}) + \beta_6 \text{DefSpread}_t$, using a stepwise procedure. Beneath the variables, in parenthesis, we report significance values. n. e. means not entered in the regression.

<table>
<thead>
<tr>
<th></th>
<th>Merton Model</th>
<th>LS Model</th>
<th>ER Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>1.54 (0.017)</td>
<td>-0.81 (0.027)</td>
<td>-0.88 (0.011)</td>
</tr>
<tr>
<td>$r_{t}^{10}$</td>
<td>n. e.</td>
<td>n. e.</td>
<td>n. e.</td>
</tr>
<tr>
<td>$(r_{t}^{10})^2$</td>
<td>n. e.</td>
<td>n. e.</td>
<td>n. e.</td>
</tr>
<tr>
<td>$r_{t}^{10} - r_{t}^2$</td>
<td>n. e.</td>
<td>80.78 (0.007)</td>
<td>88.10 (0.002)</td>
</tr>
<tr>
<td>$\sigma_{S&amp;P}$</td>
<td>-8.00 (0.030)</td>
<td>n. e.</td>
<td>n. e.</td>
</tr>
<tr>
<td>$R_{M} - r_{t}^{1m}$</td>
<td>-2.59 (0.025)</td>
<td>n. e.</td>
<td>-3.70 (0.001)</td>
</tr>
<tr>
<td>DefSpread</td>
<td>n. e.</td>
<td>n. e.</td>
<td>n. e.</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.56</td>
<td>0.43</td>
<td>0.52</td>
</tr>
<tr>
<td>$F$</td>
<td>9.12</td>
<td>10.62</td>
<td>15.17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Merton Model</th>
<th>LS Model</th>
<th>ER Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(0.005)</td>
<td>(0.007)</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>(0.000)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.002)</td>
<td>(0.001)</td>
</tr>
</tbody>
</table>
Figure 1 - EDPs of Bankrupt Companies

Figure 2 – Distribution of EDPs: Merton’s Model
Figure 3 – Distribution of EDPs: Longstaff and Schwartz Model

Figure 4 – Distribution of EDPs: Ericsson and Reneby Model