EVENT STUDIES WITH CONDITIONALLY HETEROSCEDASTIC STOCK RETURNS

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Abstract:

Empirically, we show that the proportion of stocks exhibiting conditional heteroscedastic residuals, is high. We suggested to use the market model with GARCH(1,1) residuals in order to describe daily stock returns and derived a test statistics for the null hypothesis of no abnormal returns, which is an extension of Boehmer et al. (1991) test statistics.

Monte Carlo simulations and simulations on real data show that the test statistics accounting for conditional heteroscedasticity, Boehmer et al. (1991) test and the Generalized Sign test proposed by Cowan (1992) are well specified. The test statistics accounting for conditional heteroscedasticity dominates the previous tests in terms of power. Interestingly, Monte Carlo simulations and simulations on real data lead to very close results concerning the specification and the power of the test statistics so that the market model with GARCH (1,1) residuals can be seen as a reasonable approximation of the true data generating process.

Keywords: Event studies, Market Model, GARCH

JEL Classifications: G14, C10
1. Introduction

Since Fama, Jensen, Fisher and Roll (1969), the market model which relate linearly the individual stock returns to the market returns is used routinely to gauge daily abnormal returns (the difference between forecasted returns and realized returns) around a firm-specific event. The construction of an hypothesis test of the null (i.e. no abnormal returns), involves additional hypotheses however, the most common being independent and identically normal errors terms. Nevertheless, the behavior of conditional second moment of the market model has not been examined in great details.

The introduction of ARCH class models by Engle (1982) and Bollerslev (1986) has been used extensively and successfully to modeled the conditional variance of portfolio stock returns; see the surveys of Bollerslev, Chou and Kroner (1992), Bera and Higgins (1993), Bollerslev, Engle and Nelson (1994) and Palm (1995) among others. However, the innovations in individual daily stock returns series are frequently long live or permanent. Lamoureux and Lastrapes (1990) find that the conditional heteroscedasticity of individual stock returns almost disappears when it is expressed as a function of the past trading volumes. Bera, Bubnys and Park (1988) show that the conditional variance of the index returns does not subsume the total conditional variance of individual stock returns, or stated in other words, the residuals of the market model itself exhibit conditional heteroscedastic variance.

Cowan (1992) and Campbell and Wasley (1993) report substantial departure from normality in the abnormal returns of NASDAQ stocks. In particular, abnormal returns are found to have a high marginal kurtosis. These facts are is important for correctly
testing the null hypothesis of no abnormal returns in small samples. They suggests that
the series may exhibit conditional heteroscedasticity too. The presence of conditional
heteroscedasticity may lead to ill-specified and/or less powerful tests in small samples.
Brockett, Chen and Garven (1999) and Hilliard and Savickas (2001) show that the
conclusions of an event-study can be reversed in this context.

Several papers have proposed to explicitly account for heterosecdastic residuals. For
example, De Jong, Kemna and Kloeck (1992) developed a extension of the market
model which allows to test for any periodic event. In particular, they show that the errors
of the market model are conditionally heteroscedastic and that a GARCH(1,1)
specification provides an adequate parsimonious representation of the conditional
variance. Kryzanowski and Zhang (1993) assume that individual stock returns are
described by the market model which residuals are ARCH (q). With both methods, it is
important to note that the one-day abnormal returns and its confidence interval are
estimated directly by introducing a dummy variable. The extension of these methods to
longer windows, which is required to estimate the valuation effect of an event, is not
straightforward.

Jayanti and Booth (1992), Brockett, Chen and Garven (1994), Corhay and Tourani
Rad (1996) estimate the parameters of the market model with conditionally
heterosecdastic residuals over a period prior to the event. Then, the model is used it to
forecast the conditional mean and the conditional variance over the event-window. With
the exception of one step-ahead forecasts, this method produces misleading results
because of biased confidence intervals. Baillie and Bollerslev (1992) show that the
knowledge of the conditional variance at the forecast horizon $s, (s > 1)$ is not sufficient
to construct the confidence interval for the conditional mean even when the errors are conditionally normal. The conditional distribution of the forecasted errors has a excess kurtosis. In order to obtain the correct confidence interval for conditionally gaussian GARCH (1,1) models, the authors propose a correction based on Cornish-Fisher expansion. However, the hypothesis consisting in testing the null of temporally aggregated abnormal returns (cumulated abnormal returns) is intractable. Another problem related to this approach is that both the conditional and the unconditional second moments are assumed to remain constant after the event. This is quite restrictive as some events are known to modify the unconditional variance; see for instance in the case of stock splits.

Recently, Hilliard and Savickas (2001) have examined the specification and the power of various test statistics under the null of no abnormal returns in which the residuals of the market model are assumed to be GARCH(1,1). The simulations show that a test statistics accounting for conditional heteroscedasticity has a higher power than traditional tests. However, these are based on the assumption that the forecasts are conditionally normal making their results questionable for longer horizons. Savickas (2002) proposes to circumvent this assumption by estimating a similar model using dummy variables over the event-period. Simulations show also a substantial gain in power for the test statistics adjusted for conditional heteroscedasticity.

In this paper, we pursue two objectives. First, we examine the residuals of the market model and determine which stocks are prone to exhibit conditional heteroscedastic residuals. Second, we extend the model in order to test the null of no abnormal returns and no cumulated abnormal returns over a fixed window which is stock specific. We
develop a test statistics which specification and power are examined using Monte-Carlo simulations and Brown and Warner (1980, 1985) simulations based on real data. We check the robustness of the test when a jump in the unconditional variance is artificially introduced.

Our main results can be summarized as follows. First, based on randomly selected stocks, we show that 35% (42%) of NYSE-AMEX (NASDAQ) stocks exhibit conditional heteroscedastic residuals. Second, and more surprisingly, this proportion decreases slightly with trading volume from 37% (46%) in the highest quintile stock to 31% (33%) in the lowest quintile. Third, we propose a modified version of the BMP statistics to account for conditional heteroscedastic errors. We show that this new test statistics is well-specified for testing the null of no abnormal returns (cumulated abnormal returns). This test is more powerful than the original parametric test proposed by Boehmer, Musumeci and Poulsen (1991) and the non parametric test proposed by Cowan (1992).

The remainder of the paper is organized as follows. We examine the residuals of the market model adjusted or not for infrequent trading conditional on firm’s liquidity in Section 2. In Section 3, the experimental design is described. In Section 4, we study the specification and the power of the test statistics using simulations. Section 5 summarizes our main results.

2. Do Market Model residuals exhibit conditional heteroscedasticity?

2.1. Simulation design

In this analysis we use all the NYSE/AMEX and NASDAQ firms with available data on the Daily CRSP files. The period covered goes from July 1962 through December
1998 for the NYSE/AMEX and from January 1973 through December 1998 for the NASDAQ. Financial research focuses mainly on ordinary common shares so that CRSP share codes 10 and 11 are eliminated from our analysis. We use the Daily Files to compute logarithmic daily returns adjusted for capital changes and dividends. The market returns are computed from the value weighted CRSP index.

As liquidity is measured differently for NYSE-AMEX and NASDAQ, stocks are considered separately according to the market on which they are listed. The sampling is conducted in two steps. First, we select randomly a NYSE-AMEX (NASDAQ) stocks. Second, we draw randomly a date between 07/62-12/97 (01/73-12/97) and consider the returns of the selected firm over a period of 260 days beginning on the selected date. Whenever the stock is not continuously listed during the selected period, a new stock is drawn. This ensures that the probability of being selected is higher for stocks which are listed during a longer period. This procedure is repeated 10 000 times. The volume expressed in USD and the number of days for which the number of shares traded is nil are also collected for each stock.

We estimate the constant mean return model, the standard market model and the Scholes and Williams market model with 5 leads-lags and 10 lead-lags over the 260-days period. For each stock, we compute the residuals of these models. The null hypothesis of no conditional heteroscedasticity within the residuals is tested with the Lagrange Multiplier test (LM test hereafter) with 5 and 10 lags; see Engle (1982). To be more specific, we estimate the following regression:

\[ e_t^2 = c_0 + c_1 e_{t-1}^2 + \ldots + c_p e_{t-p}^2 + \eta_t \]

where
e’s are the residuals of the conditional mean estimated by OLS,

\( T \) is the sample size (260) and \( \eta_i \) is an error term,

\( R_u^2 \) the uncentered coefficient of determination of the regression.

Under the null hypothesis of no ARCH effects (i.e. \( c_1 = \cdots = c_p = 0 \)), the quantity \( TR_u^2 \) is asymptotically distributed as a chi-squared with \( p \) degrees of freedom. Finally, we compute the proportion of firms for which the null (no conditional heteroscedasticity) is rejected.

2.2. The results

- Global results

We estimate the residuals with two different benchmarks: the constant mean return model and the market model. For the latter, we use the OLS estimators as well as the Fowler and Rorke (1983) estimators to account for non-synchronous trading. The proportion of stocks exhibiting conditional heteroscedastic market model residuals is presented in Table 1. Three main results are worth mentioning. First, roughly speaking, one stock over three presents conditional heteroscedastic residuals. The market model helps little in reducing (from 35.29% to 33.29%, LM test with 5 lags) the rejection of the null. Second, the rejection rate of the null is higher (two stocks over five) for NASDAQ stocks. Third, the rejection rate decreases from 33.39% (40.46%) to 27.08% (35.96%) for NYSE-AMEX (NASDAQ) stocks after adjusting the market model parameters for thin trading. Overall, the proportion of stocks with conditional heteroscedastic residuals is not especially high but the potential link between infrequent trading and conditional heteroscedasticity deserves a closer examination.

Insert Table 1
Conditional heteroscedasticity and on liquidity

We divide the sample in quintiles based on trading volume expressed in dollars (\( Q_{1,\text{vol}} \) corresponds to low volume) and the number of days with no transaction (\( Q_{1,\text{no-trans}} \) corresponds to a high number of days with no transactions). As shown in Table 2, Panel A, the rejection rate of the null for NYSE-AMEX stocks is a decreasing function of the liquidity, the difference between quintile 1 and quintile 5 lies between 8.95% (trading volume in dollars and market model) and 12.65% (no trading volume and Scholes and Williams adjustment with 10 leads-lags). Most of this reduction occurs between the quintile 1 and quintile 2.

For NASDAQ stocks (see Table 2, Panel B), the decrease ranges from 15.4% (trading volume in dollars and market model) and 17.10% (no trading volume and Scholes and Williams adjustment with 10 leads-lags). The main difference with NYSE stocks lies in the fact that the reduction is not specific to any quintile.

3. Experimental Design

3.1. The model

- The norm
Daily stock returns are assumed to follow the market model with conditional heteroscedastic residuals. To be more specific:

\[ r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_{it} \]

\[ \varepsilon_{it} \mid \varepsilon_{it-1}, \varepsilon_{it-2}, \ldots \sim N(0, h_{it}) \]

\[ h_{it} = a_{i0} + a_{i1}\varepsilon_{it-1}^2 + a_{i2}h_{it-1} \]

where

\( r_{it} (r_{mt}) \) is the continuously compound daily stock (market) returns adjusted for dividend and capital changes at time \( t \),

\( \varepsilon_{it} \) are the residuals of the market model,

In order to keep the estimation tractable and to insure a covariance-stationary process, we impose further restrictions:

\( \varepsilon_{it} \) and \( \varepsilon_{jt} \) are independent of each other for every \( i, j, t \) and \( t \) whenever \( i \neq j \),

\[ a_{i0} > 0, a_{i1} \geq 0, a_{i2} \geq 0 \text{ and } a_{i1} + a_{i2} \leq 1 \]

A restricted version of this model (\( a_{i1} = 0 \) and \( a_{i2} = 0 \)) is routinely used in empirical studies and in simulations; see Brown and Warner (1985), Boehmer et al. (1991), Cowan (1992) and Cowan and Sargeant (1996) among others.

As GARCH(1,1) can be view as a parsimonious representation ARCH (p) processes, it is an extension of Krysanowski and Zhang (1993). De Jong et al. (1992) consider a more complex model in which the beta follows a random walk, daily stock returns are ARMA (1,1) and the errors are distributed according a Student-t distribution. The number of parameters to estimate is high (a minimum of ten compare to five in our

\(^1\)Note that Brockett et al. (1999) do not impose such restrictions. In fact, 11 of the firms over 20 violate the covariance-stationary conditions.
model) which requires a number of observations (921 in their case) exceeding what is usually employed in the event-study literature (250). Moreover, Bera and Higgins (1993) show that a regression with time-varying beta can also be represented as a standard regression with conditional heteroscedastic residuals so that our model is expected to capture this effect.

- Abnormal returns
  As in De Jong et al. (1992) and Kryzanowski and Zhang (1993), the abnormal returns are estimated by adding dummy variables into the model in the following way:

\[ r_{it} = \alpha_i + \beta_i r_{mt} + \sum_{k=T_1}^{T_2} y_{ik} \delta_{ik} + \epsilon_{it} \]

\[ \epsilon_{it} | \epsilon_{it-1}, \epsilon_{it-2}, \ldots \sim N(0, h_{it}) \]

\[ h_{it} = a_{i0} + a_{i1} \epsilon_{it-1}^2 + a_{i2} h_{it-1} \]

where

- \( y_{ik} \) is the abnormal return estimated for the firm \( i \) on day \( k \),
- \( \delta_{ik} \) is a dummy variable which takes the value 1 when \( t = k \) and 0 otherwise,
- \( T_1 \) (\( T_2 \)) is the beginning (end) of the window.

The main advantage of this model is to obtain simultaneously the expected value of the abnormal returns and their standard deviations which allows for a direct test of their significance.

- Cumulated abnormal returns
  Unfortunately, the confidence interval for the cumulated abnormal returns cannot be derived easily from the abnormal returns (mean and standard deviation). To gauge the global effect of the event, the following model is estimated:
\( r_i = \alpha_i + \beta_i r_{mt} + \Gamma_i \delta_{[T_i^1;T_i^2]} + \epsilon_i \)
\( \epsilon_i \big| \epsilon_{i-1}, \epsilon_{i-2}, \ldots \sim N(0, h_i) \)
\( h_i = a_{i0} + a_{i1} \epsilon_{i-1}^2 + a_{i2} h_{i-1} \)

where

\( \Gamma_i \) is the cumulated abnormal returns over the period \([T_i^1;T_i^2]\) surrounding the event,

\( \delta_{[T_i^1;T_i^2]} \) is a dummy variable which takes the value \( 1/(T_i^2 - T_i^1 + 1) \) when \( t \in [T_i^1;T_i^2] \) and 0 otherwise.

This specification provides the cumulated abnormal returns and its standard deviation directly.

- Changes in the unconditional variance

The unconditional variance of the residuals is:

\( \sigma_{\epsilon_i}^2 = \frac{a_{i0}}{1 - a_{i1} - a_{i2}} \)

Any change in the unconditional variance can be caused by a change in either of the parameters describing the conditional variance. In order to examine such changes, we study the following specification:

\( r_{it} = \alpha_i + \beta_i r_{mt} + \sum_{k=T_i^1}^{T_i^2} \gamma_{ik} \delta_{ikt} + \epsilon_{it} \)
\( \epsilon_{it} \big| \epsilon_{it-1}, \epsilon_{it-2}, \ldots \sim N(0, h_i) \)
\( h_i = a_{i0} + \Delta a_{i0} D_{it} + a_{i1} \epsilon_{it-1}^2 + \Delta a_{i1} \epsilon_{it-1}^2 D_{it} + a_{i2} h_{it-1} + \Delta a_{i2} h_{it-1} D_{it} \)

where

\( D_{it} \) is a dummy variable which is 0 before the event and 1 after.
To insure stationnarity, we impose the following restrictions: \( a_{i0} + \Delta a_{i0} > 0 \), \( a_{i1} + \Delta a_{i1} \geq 0 \), \( a_{i2} + \Delta a_{i2} \geq 0 \) and \( a_{i1} + \Delta a_{i1} + a_{i2} + \Delta a_{i2} < 1 \).

After the event, the unconditional variance becomes:

\[
\sigma_{after, \epsilon_i}^2 = \frac{a_{i0} + \Delta a_{i0}}{1 - a_{i1} - \Delta a_{i1} - a_{i2} - \Delta a_{i2}}
\]

This specification is designed to capture a permanent jump in the unconditional variance while the temporary effects are assumed to behave as a GARCH process. Temporary jumps in the unconditional variance as in Giaccotto and Sfiridis (1996) are not examined because these temporary jumps try to mimic changes in the conditional variance which is exactly what GARCH residuals are designed for.

3.2. Hypothesis testing

- The null

Basically, the standard hypothesis states that the event has no impact on the behaviour of the mean return at time \( t \) (\( t \) being a day within the window) so that the average abnormal return is nil. More formally, we write:

\[
H_{0, \epsilon_i} : \frac{1}{N} \sum_{i=1}^{N} \gamma_{it} = 0 \quad \text{vs} \quad H_{A, \epsilon_i} : \frac{1}{N} \sum_{i=1}^{N} \gamma_{it} \neq 0
\]

The cumulative effect of the event is aggregated through time at the firm level. The valuation effect of the event is tested cross-sectionally as follows:

\[
H_{0} : \frac{1}{N} \sum_{i=1}^{N} \Gamma_{ti} = 0 \quad \text{vs} \quad H_{A} : \frac{1}{N} \sum_{i=1}^{N} \Gamma_{ti} \neq 0
\]

- Test statistics
\[
J_t = \frac{\sum_{i=1}^{N} \hat{SRC}_it}{\sqrt{N\hat{\sigma}_{cross_it}}}
\]

where

\[
\hat{SRC}_it = \frac{\hat{\gamma}_it}{\hat{\sigma}(\hat{\gamma}_it)}
\]

is the standardized abnormal returns,

\[
\hat{\sigma}_{cross_it}
\]

is the standard deviation of \( \hat{SRC}_it \).

Under the null hypothesis of no abnormal returns, the \( J_t \) statistics is distributed as a Student-t variable with \( N-1 \) degrees of freedom.

**3.3. Estimation**

The parameters of the model are estimated by maximizing the log-likelihood function:

\[
L_T = \frac{1}{T} \sum_{t=1}^{T} l_t(\theta)
\]

where

\[
l_t(\theta) = \frac{1}{2} \log(h_{it}) - \frac{1}{2} \epsilon_{it}^2 h_{it}^{-1}
\]

and \( \theta = (\alpha, \beta, \gamma_{it1}, \ldots, \gamma_{itK}, a_0, a_i, a_{2i}) \).

The variance-covariance matrix of the quasi-maximum likelihood estimators is derived analytically with respect to the parameters and estimated:

\[
\hat{I} = \frac{1}{T} \sum_{t=1}^{T} \frac{\partial l_t}{\partial \theta} \frac{\partial l_t}{\partial \theta'} \quad \text{and} \quad \hat{J} = -\frac{1}{T} \sum_{t=1}^{T} \frac{\partial^2 l_t}{\partial \theta \partial \theta'}
\]

Then even when \( \frac{\epsilon_{it}}{\sqrt{h_{it}}} \) is actually non-gaussian, we still have the asymptotic property:

\[
\sqrt{T} \left( \hat{\theta} - \theta \right) \overset{L}{\rightarrow} N \left( 0, \hat{J}^{-1} \hat{J} \right)
\]
We use BHHH algorithm\(^2\) to calculate the maximum of the log-likelihood function. The initial values correspond to the OLS regression coefficients for the mean equation
\[ \hat{b}_0 = (\hat{\alpha}_0, \hat{\beta}_0, \hat{\gamma}_{0T}, \ldots, \hat{\gamma}_{0T_f}) \] while \( \hat{\omega}_0 \) is set equal to \((\hat{\sigma}_\varepsilon, 0, 0)\) where \( \hat{\sigma}_\varepsilon \) is the unconditional variance of the residuals. We do not try several starting points for the GARCH equation because the estimation itself is highly time-consuming. A wrong starting point could explain why the algorithm fails to converge in some cases.

4. Simulations

First, we examine the specification and the power of the test statistics under a perfectly controlled sample design (Monte-Carlo). Our objective is to gauge the improvement (if any) of the model in case of conditional heteroscedastic residuals. Second, we use simulations a la Brown and Warner in order to analyze the real impact of the method when the residuals are and are not conditionally heteroscedastic.

4.1. Monte-Carlo simulations

Each series contains 260 observations, the last 10 of which refer to the event-window. We calibrate the model as in Hilliard and Savickas (2001) in order to make the results comparable. The simulation of the market model is not required in this setting so that the parameter \( \alpha_i \) is drawn randomly from a uniform distribution \([5 \times 10^{-4}, 3.5 \times 10^{-3}]\).

We consider three different cases corresponding to no ARCH dependence \((a_i = 0)\), moderate dependence \((a_i = 0.45)\), and strong dependence \((a_i = 0.90)\). The autoregressive component \( a_{i2} \) is drawn randomly from the uniform distribution \([0;1-a_i]\) in
order to obtain a stationary process. Finally, $a_{i0}$ is drawn from the uniform distribution

$$\begin{bmatrix}
\frac{0.2(1-a_{11}-a_{12})}{365}, & \frac{0.6(1-a_{11}-a_{12})}{365}
\end{bmatrix}$$

For simplicity, we simulate the jump in the unconditional variance by means of a change in the constant term of the conditional variance equation. The jump occurs the first day of the window for the cumulated abnormal returns and on the fifth day otherwise.

4.1.1 Cumulated abnormal returns

- Specification

The results are presented in Table 3. The $J$ adjusted t-stat and the BMP adjusted t-stat are well-specified. Both tests are robust to highly dependant conditional second moments and to jumps in the unconditional variance. Unlike previous tests, the rejection rate of the generalized sign test is too low in the presence of conditional heteroscedastic residuals. The same results applies to the jumps in the unconditional variance confirming Giaccoatto and Sfidiris (1996) results. We suspect this test to be sensitive to time-series dependence or to non stationary series (i.e. jump in the unconditional variance). In this framework, the generalized sign test must be considered with caution.

Insert Table 3

- Power

In this setting, we add a cumulated abnormal returns of 2% to the series during the event-window. We assume the abnormal returns to be uniform over time ($\gamma_{ik} = 0.2\%$).

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2 We use the CML library of GAUSS Software. In order to increase the speed of convergence and the precision, the gradient is defined analytically.
Table 4 show that the $J$ adjusted t-test is the most powerful. The generalized sign test has the lowest power in all the cases we examine so that we restrict the comparison to the $J$ and BMP adjusted t-test. Not surprisingly, the gain in using our test statistics is particularly important when ARCH effects are present. For example, our test detects cumulated abnormal returns in 94.80% of the cases while 52.80% cumulated abnormal returns are found with the BMP test. When a jump in the unconditional variance is introduced, the gain decreases slightly but it is still in favor of the $J$ adjusted t-test. When the unconditional variance is doubled, the power of the $J$ test statistics is still 83.20% (at the 5% level) against a low 39.20% for the BMP test statistics. Conversely, the power of both tests increases to 97.60% and 65.20% respectively when the unconditional variance is reduced by half.

4.1.2 Abnormal returns

We examine the specification of the test statistics for the five-days horizon. The results are presented in Table 5. As previously, the parametric tests are well-specified. The non-parametric generalized sign test strongly understates the rejection rate when the unconditional variance decreases and the conditional heteroscedasticity is high ($a_t = 0.90$)

Insert Table 5

We study the power of the test statistics by adding a 1% abnormal returns on day 254 (the event-day). Table 6 shows that the conditional test is the most powerful in detecting abnormal returns with a power close to 100% when ARCH effects are stronger. In that setting, the classic test as a strong power too because the relative magnitude of the unconditional jump is lower when the conditional variance is high. However, the power
of the tests (including the $J$ adjusted t-test) decreases dramatically when a jump occurs with a conditionally homoscedastic series. Stated in other words, all the tests presented here have low power to detect abnormal returns when the series are homoscedastic outside the window and the event induces a jump in the variance.

Insert Table 6

4.2. Simulations on real data

The simulations are conducted as in Section 2.2. on NYSE-AMEX stocks. 250 portfolios consisting of 50 stocks are constructed. For each stock, we estimate the market model parameters and the residuals via an OLS regression with 260 daily returns. We calculate the $TR^2_u$ statistics with five lags and sort the stocks in two groups (ARCH/no-ARCH stocks) according to its value, the breakpoint being the 5% level. Finally, we estimate the market model with GARCH(1,1) errors. On average, the BHHH algorithm fails to converge for 15% (10%) of the stocks within the ARCH group (non-ARCH group). In that case, a new stock is drawn$^3$.

Concerning the simulation of the jump in the unconditional variance, Brown and Warner (1985) and Boehmer et al. (1991) suggested methods which are not designed for time-dependent conditional second moments. As in Hilliard and Savickas (2001), we multiply the abnormal returns by the square root of the jump magnitude $\left(\sqrt{\lambda}\right)$. In a first pass, we estimate the model with the 260 observations. Second, the residuals within the

$^3$ Hilliard and Savickas (2001) find that the algorithm fails to converge for 10% of the firms. This fact is not mentioned in literature, however it is of concern because it shows how well the model fits the data.
window are multiplied by $\lambda$ leading to a new series. Third, we estimate the market model with GARCH (1,1) residuals for the new series.

- Specification

The results concerning the specification of the test are presented in Table 7. The replication of the simulations designed to estimate the specification of the test statistics proposed by Boehmer et al. (1991) method and Cowan (1992) show similar results to those previously found in the case of no change in variance. Other simulations are not directly comparable because we do not simulate the jump in variance as previously done.

Independently of the behavior of both the conditional and unconditional variance within the window, all the test statistics we examine are well-specified. Moreover, there is no major difference between those stock exhibiting strong conditional second moment dependences and those who do not.

Insert Table 7

- Power

The results concerning the power of the tests are presented in Table 8. They confirm what was found with Monte-Carlo simulations. The J test is the most powerful of the three tests we examine. Nevertheless, the power of the test is really low, in particular in the no-ARCH case (36.4% at the 5% level with no jump in the variance). The power of the test deteriorates even more when in the unconditional variance experiments a jump. The sample size must be substantially higher in order to obtain a reasonable power for the test statistics.
4.2.2. Abnormal returns

In Table 9, the results concerning the specification (Panel A) and the power (Panel B) of the tests are presented. As it was found in the previous setting, the three test statistics are well-specified. Thus, detecting a significant market reaction when there is none is not of major concern in empirical studies.

Concerning the power of the tests, the J test which accounts for conditionally heteroscedastic residuals performs better. It dominates clearly the BMP and the Generalized sign test. However, as noted for the cumulated abnormal returns the power of these tests, including the J test, is alarmingly low for the no-ARCH sub-group when there is an increase in the unconditional variance.

4.3. Discussion

5. Conclusion

We investigated the behavior of the residuals of the constant mean return model, the market model and the market model adjusted for thin trading. We showed that, on average, one stock over three exhibit conditional heteroscedastic residuals. This proportion increases to one stock over two for thinly traded stocks listed on the NASDAQ. However, this fact is generally ignored in event-studies. Thus, we addressed the question of how the conditional heteroscedasticity affects the specification and the power of test statistics that are routinely used.
We suggested to use the market model with GARCH(1,1) residuals in order to describe daily stock returns and derived a test statistics for the null hypothesis of no abnormal returns, which is an extension of Boehmer et al. (1991) test statistics. This model can easily handle jumps in the unconditional variance during the event-period (window).

Monte Carlo simulations and simulations on real data show that the test statistics accounting for conditional heteroscedasticity, Boehmer et al. (1991) test and the Generalized Sign test proposed by Cowan (1992) are well specified. In short, there is little risk to detect wrongly a non-significant event. However, the test statistics accounting for conditional heteroscedasticity dominates the previous tests in terms of power. This means that events previously considered as neutral in terms of valuation may affect the value of the firm. Interestingly, Monte Carlo simulations and simulations on real data lead to very close results concerning the specification and the power of the test statistics so that the market model with GARCH (1,1) residuals can be seen as a reasonable approximation of the true data generating process.
References


Table 1: Proportion (in %) of Stocks Exhibiting Significant ARCH Effects

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<th>NYSE – AMEX</th>
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<td>ARCH10</td>
</tr>
<tr>
<td>Constant Mean Return</td>
<td>35.29</td>
<td>30.51</td>
<td>41.88</td>
<td>37.32</td>
</tr>
<tr>
<td>Market Model</td>
<td>33.39</td>
<td>27.90</td>
<td>40.46</td>
<td>35.80</td>
</tr>
<tr>
<td>SW 5</td>
<td>30.17</td>
<td>24.75</td>
<td>38.45</td>
<td>33.94</td>
</tr>
<tr>
<td>SW 10</td>
<td>27.08</td>
<td>22.45</td>
<td>35.96</td>
<td>31.67</td>
</tr>
</tbody>
</table>

We report the proportion of stocks which residuals exhibit significant conditional heteroscedasticity at the 5% level based on the LM test with 5 (ARCH5) and 10 lags (ARCH10). 10,000 stocks are drawn randomly over a period of 250 consecutive days. Three models are used to compute the residuals:

\[
\hat{\epsilon}^i = r_i - \hat{\mu}, \quad \hat{\epsilon}^2 = r_i - \hat{\alpha} - \hat{\beta}_r r_{sw}, \quad \hat{\epsilon}^3 = r_i - \hat{\alpha}_{sw} - \hat{\beta}_{sw} r_{sw}
\]

where \(\hat{\mu}\) is the sample mean, \(\hat{\alpha}\) and \(\hat{\beta}_r\) are the OLS estimates, and \(\hat{\alpha}_{sw}\) and \(\hat{\beta}_{sw}\) are the Scholes and Williams estimates with 5 (10) leads-lags.

Table 2: Proportion of Stocks Exhibiting ARCH Effects Conditional on Stock Trading Intensity

<table>
<thead>
<tr>
<th></th>
<th>Volume Dollar</th>
<th>No Transaction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1  2  3  4  5</td>
<td>1  2  3  4  5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Panel A: NYSE and AMEX</td>
</tr>
<tr>
<td>Constant Mean Return</td>
<td>39.85 33.55 36.95 35.15 30.95</td>
<td>40.00 34.90 35.05 35.45 31.05</td>
</tr>
<tr>
<td>Market Model</td>
<td>39.95 31.25 33.65 31.10 31.00</td>
<td>39.75 31.80 32.60 31.75 31.05</td>
</tr>
<tr>
<td>SW 5</td>
<td>36.65 29.55 30.05 28.30 26.30</td>
<td>37.35 29.95 29.40 27.80 26.35</td>
</tr>
<tr>
<td>SW 10</td>
<td>34.80 26.60 26.50 24.40 23.10</td>
<td>35.80 27.30 24.80 23.45 23.15</td>
</tr>
</tbody>
</table>

We report the proportion of stocks which residuals exhibit significant conditional heteroscedasticity at the 5% level based on the LM test with 5 (ARCH5) and 10 lags (ARCH10) for the NYSE-AMEX (Panel A) and NASDAQ (Panel B) stocks. The sample is divided into quintiles according to two criteria: daily average trading volume expressed in USD and the number of days for which the trading volume is nil.
We simulate the following model for 250 portfolios of 50 series each:

\[
R_t = \alpha + \Gamma \delta_t + \varepsilon_t, \\
\varepsilon_t | \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots \sim N(0, h_t) \\
h_t = a_0 + a_1 \varepsilon_{t-1}^2 + a_2 h_{t-1}
\]

where \( \Gamma_t \) is the cumulated abnormal returns over the period \([251;260]\) surrounding the event, \( \delta_{[251;260]} = 0.10 \) when \( t \in [251;260] \) and 0 otherwise.

The series have 260 observations. The coefficients \( a_{j2} \) and \( a_{0} \) are drawn randomly from a uniform distribution \([0;1-a_t]\) and

\[
\begin{bmatrix}
0.2(1-a_{01}-a_{21}) \\
0.6(1-a_{01}-a_{21})
\end{bmatrix}
\]

respectively. We report the specification of three test statistics:

a) \( J \) adjusted t-stat
b) BMP adjusted t-stat
c) Generalized sign test

when the unconditional variance experiments no change, 50%, 200% increases and a 50% decrease.

<table>
<thead>
<tr>
<th>( a_t )</th>
<th>( J \text{ test} ) ( \Delta a_0 = 0 )</th>
<th>( \text{BMP test} ) ( \Delta a_0 = 0.5a_0 )</th>
<th>( \text{G-sign test} ) ( \Delta a_0 = a_0 )</th>
<th>( \Delta a_0 = -0.5a_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.6</td>
<td>5.6</td>
<td>9.6</td>
<td>0.8</td>
</tr>
<tr>
<td>0.45</td>
<td>2.0</td>
<td>5.2</td>
<td>9.6</td>
<td>0.8</td>
</tr>
<tr>
<td>0.90</td>
<td>0.8</td>
<td>3.2</td>
<td>9.6</td>
<td>0.8</td>
</tr>
<tr>
<td>0.00</td>
<td>0.8</td>
<td>6.8</td>
<td>12.4</td>
<td>1.6</td>
</tr>
<tr>
<td>0.45</td>
<td>0.8</td>
<td>5.2</td>
<td>10.0</td>
<td>1.2</td>
</tr>
<tr>
<td>0.90</td>
<td>0.8</td>
<td>4.4</td>
<td>9.2</td>
<td>0.4</td>
</tr>
<tr>
<td>0.00</td>
<td>1.2</td>
<td>6.4</td>
<td>11.2</td>
<td>1.2</td>
</tr>
<tr>
<td>0.45</td>
<td>1.2</td>
<td>4.4</td>
<td>8.0</td>
<td>0.4</td>
</tr>
<tr>
<td>0.90</td>
<td>0.4</td>
<td>3.6</td>
<td>7.2</td>
<td>0.8</td>
</tr>
<tr>
<td>0.00</td>
<td>2.0</td>
<td>4.0</td>
<td>10.8</td>
<td>0.8</td>
</tr>
<tr>
<td>0.45</td>
<td>0.8</td>
<td>3.2</td>
<td>10.4</td>
<td>0.4</td>
</tr>
<tr>
<td>0.90</td>
<td>0.4</td>
<td>4.0</td>
<td>11.2</td>
<td>0.0</td>
</tr>
</tbody>
</table>
### Table 4: Power of the Test Based on Cumulated Abnormal Returns with Monte-Carlo simulations

<table>
<thead>
<tr>
<th>$a_i$</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
<th>J-test</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
<th>BMP test</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
<th>G-sign test</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>7.6</td>
<td>26.0</td>
<td>39.2</td>
<td>8.4</td>
<td>22.4</td>
<td>33.2</td>
<td>6.4</td>
<td>18.8</td>
<td>28.8</td>
<td>0</td>
<td>0</td>
<td>0.00</td>
<td>0.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.45</td>
<td>26.0</td>
<td>46.8</td>
<td>54.4</td>
<td>6.0</td>
<td>20.4</td>
<td>32.8</td>
<td>4.0</td>
<td>19.2</td>
<td>30.0</td>
<td>0</td>
<td>0</td>
<td>0.45</td>
<td>0.45</td>
<td>0</td>
<td>0.45</td>
</tr>
<tr>
<td>0.90</td>
<td>89.2</td>
<td>94.8</td>
<td>96.4</td>
<td>30.8</td>
<td>52.8</td>
<td>65.2</td>
<td>16.0</td>
<td>38.8</td>
<td>50.4</td>
<td>0</td>
<td>0</td>
<td>0.90</td>
<td>0.90</td>
<td>0</td>
<td>0.90</td>
</tr>
</tbody>
</table>

We simulate the following model for 250 portfolios of 50 series each:

$$R_i = \alpha_i + \Gamma_i \delta_{[251;260]} + \epsilon_i \epsilon_{i-1}; \epsilon_{i-2}, \ldots \sim N(0, \sigma^2)$$

where $\Gamma_i$ is the cumulated abnormal returns over the period [251;260] surrounding the event, $\delta_{[251;260]} = 0.10$ when $t \in [251;260]$ and 0 otherwise.

The series have 260 observations and a 0.2% daily abnormal returns is added for each day in the window (2% cumulated abnormal return). The coefficients $a_{12}$ and $a_{10}$ are drawn randomly from a uniform distribution $[0; a_{12}]$ and $[0.2(1-a_{12})]$, respectively.

We report the power of three test statistics:

- **a)** J-adjusted t-stat
- **b)** BMP adjusted t-stat
- **c)** Generalized sign test

when the unconditional variance experiments no change, 50%, 200% increases and a 50% decrease.
Table 5: Specification of the Test Based on Abnormal Returns (5 days) with Monte-Carlo simulations

<table>
<thead>
<tr>
<th>$a_i$</th>
<th>$J_{test}$ 1% 5% 10%</th>
<th>BMP test 1% 5% 10%</th>
<th>G-sign test 1% 5% 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta a_0 = 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>0.8 5.2 11.6</td>
<td>1.2 4.0 12.0</td>
<td>0.4 4.8 8.8</td>
</tr>
<tr>
<td>0.45</td>
<td>0.4 2.8 6.8</td>
<td>0.4 2.8 7.6</td>
<td>0.0 3.2 8.4</td>
</tr>
<tr>
<td>0.90</td>
<td>0.0 4.4 7.6</td>
<td>0.0 1.6 6.4</td>
<td>0.0 2.8 8.8</td>
</tr>
<tr>
<td></td>
<td>$\Delta a_0 = 0.5a_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>0.0 4.0 13.2</td>
<td>0.0 4.0 10.4</td>
<td>0.8 4.0 6.0</td>
</tr>
<tr>
<td>0.45</td>
<td>2.0 5.6 10.4</td>
<td>1.6 6.4 10.4</td>
<td>0.0 1.2 6.4</td>
</tr>
<tr>
<td>0.90</td>
<td>1.6 5.6 10.4</td>
<td>2.0 5.2 8.8</td>
<td>1.6 4.8 8.8</td>
</tr>
<tr>
<td></td>
<td>$\Delta a_0 = a_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>0.8 4.4 10.0</td>
<td>0.8 4.0 10.8</td>
<td>0.8 5.6 10.8</td>
</tr>
<tr>
<td>0.45</td>
<td>1.2 6.4 11.6</td>
<td>0.4 4.8 8.0</td>
<td>0.8 6.4 14.8</td>
</tr>
<tr>
<td>0.90</td>
<td>1.2 4.4 10.0</td>
<td>0.0 3.2 8.4</td>
<td>0.4 3.2 8.4</td>
</tr>
<tr>
<td></td>
<td>$\Delta a_0 = -0.5a_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>0.0 4.0 11.2</td>
<td>0.0 5.6 8.4</td>
<td>0.0 6.0 11.2</td>
</tr>
<tr>
<td>0.45</td>
<td>0.8 8.4 13.6</td>
<td>1.2 6.8 13.2</td>
<td>0.8 2.8 9.2</td>
</tr>
<tr>
<td>0.90</td>
<td>0.4 6.0 11.2</td>
<td>0.4 3.6 8.8</td>
<td>0.4 1.6 3.6</td>
</tr>
</tbody>
</table>

We simulate the following model for 250 portfolios of 50 series each:

$$r_i = \alpha + \beta r_{it} + \sum_{k=1}^{T} \gamma_{it} \delta_{k,t} + \epsilon_{it} ~ N(0, h_i)$$

$$h_i = a_i \epsilon_{it} + \epsilon_{it}^2$$

where $\gamma_{it}$ is the daily abnormal return of stock $i$ on day $k$ over the period [251;260] surrounding the event, $\delta_{k,t} = 1$ when $t \in [T_{1i}; T_{2i}]$ and 0 otherwise.

The series have 260 observations. The coefficients $a_{i2}$ and $a_{i0}$ are drawn randomly from a uniform distribution $[0;1 - a_i]$ and $[0.2(1-a_i-a_{i2}); 0.6(1-a_i-a_{i2})]$ respectively. We report the power of three test statistics:

- $J$ adjusted t-stat
- BMP adjusted t-stat
- Generalized sign test

when the unconditional variance experiments no change, 50%, 200% increase and a 50% decrease.
Table 6: Power of the Test Based on Abnormal Returns (5 days) with Monte-Carlo simulations

<table>
<thead>
<tr>
<th>$\alpha_i$</th>
<th>1%</th>
<th>$J$ test 5%</th>
<th>10%</th>
<th>1% BMP test 5%</th>
<th>10%</th>
<th>1% G-sign test 5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \alpha = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>34.0</td>
<td>63.6</td>
<td>72.4</td>
<td>30.4</td>
<td>54.4</td>
<td>69.2</td>
<td>18.0</td>
</tr>
<tr>
<td>0.45</td>
<td>57.6</td>
<td>80.0</td>
<td>84.0</td>
<td>26.4</td>
<td>53.6</td>
<td>67.2</td>
<td>27.2</td>
</tr>
<tr>
<td>0.90</td>
<td>99.6</td>
<td>100.0</td>
<td>100.0</td>
<td>75.2</td>
<td>84.4</td>
<td>89.2</td>
<td>93.6</td>
</tr>
<tr>
<td>$\Delta \alpha = 0.5\alpha_0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>20.8</td>
<td>39.6</td>
<td>52.4</td>
<td>19.6</td>
<td>36.0</td>
<td>46.8</td>
<td>13.2</td>
</tr>
<tr>
<td>0.45</td>
<td>37.6</td>
<td>62.0</td>
<td>74.8</td>
<td>20.0</td>
<td>39.2</td>
<td>54.4</td>
<td>17.6</td>
</tr>
<tr>
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<td>98.4</td>
<td>100.0</td>
<td>100.0</td>
<td>55.6</td>
<td>78.0</td>
<td>82.8</td>
<td>86.4</td>
</tr>
<tr>
<td>$\Delta \alpha = \alpha_0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>18.8</td>
<td>39.6</td>
<td>49.2</td>
<td>15.6</td>
<td>34.0</td>
<td>47.2</td>
<td>8.8</td>
</tr>
<tr>
<td>0.45</td>
<td>28.0</td>
<td>54.4</td>
<td>62.8</td>
<td>11.6</td>
<td>30.0</td>
<td>46.0</td>
<td>14.8</td>
</tr>
<tr>
<td>0.90</td>
<td>94.4</td>
<td>99.2</td>
<td>100.0</td>
<td>44.0</td>
<td>69.2</td>
<td>77.6</td>
<td>80.0</td>
</tr>
<tr>
<td>$\Delta \alpha = -0.5\alpha_0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>63.2</td>
<td>84.4</td>
<td>91.2</td>
<td>54.4</td>
<td>73.2</td>
<td>84.0</td>
<td>43.2</td>
</tr>
<tr>
<td>0.45</td>
<td>87.2</td>
<td>95.2</td>
<td>97.6</td>
<td>60.4</td>
<td>77.2</td>
<td>84.0</td>
<td>67.2</td>
</tr>
<tr>
<td>0.90</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>89.2</td>
<td>97.2</td>
<td>98.4</td>
<td>99.6</td>
</tr>
</tbody>
</table>

We simulate the following model for 250 portfolios of 50 series each:

$$r_i = \alpha + \beta r_{m, t} + \sum_{k=1}^{20} \gamma_{ik} \delta_{ik} + \epsilon_{it} \mid \epsilon_{t-1}, \epsilon_{t-2}, \ldots \sim N(0, h_t)$$

$$h_t = a_{1,0} + a_{1,1} \epsilon_{t-1}^2 + a_{1,2} \epsilon_{t-2}^2$$

where $\gamma_{ik}$ is the daily abnormal return of stock $i$ on day $k$ over the period [251;260] surrounding the event, $\delta_{ik} = 1$ when $t \in [T_i^1; T_i^2]$ and 0 otherwise.

The series have 260 observations and a 1% daily abnormal returns is added on day 255. The coefficients $a_i$ and $\alpha_0$ are drawn randomly from a uniform distribution $[0;1-a_i]$ and $[0.2(1-a_i-a_0), 0.6(1-a_i-a_0)]$ respectively. We report the specification of three test statistics:

- $J$ adjusted t-stat
- BMP adjusted t-stat
- Generalized sign test

when the unconditional variance experiments no change, 50%, 200% increase and a 50% decrease.
Table 7: Specification of the Test Based on Cumulated Abnormal Returns with NYSE-AMEX data

<table>
<thead>
<tr>
<th></th>
<th>J test</th>
<th>BMP test</th>
<th>G-sign test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1%</td>
<td>5%</td>
<td>10%</td>
</tr>
<tr>
<td>No ARCH</td>
<td>2.0</td>
<td>5.2</td>
<td>8.8</td>
</tr>
<tr>
<td>ARCH</td>
<td>1.2</td>
<td>4.0</td>
<td>9.2</td>
</tr>
<tr>
<td></td>
<td>λ = 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No ARCH</td>
<td>1.2</td>
<td>6.4</td>
<td>12.0</td>
</tr>
<tr>
<td>ARCH</td>
<td>0.8</td>
<td>4.8</td>
<td>9.6</td>
</tr>
<tr>
<td></td>
<td>λ = 1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No ARCH</td>
<td>1.6</td>
<td>6.0</td>
<td>11.6</td>
</tr>
<tr>
<td>ARCH</td>
<td>0.8</td>
<td>4.0</td>
<td>7.6</td>
</tr>
<tr>
<td></td>
<td>λ = 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No ARCH</td>
<td>1.6</td>
<td>4.4</td>
<td>10.4</td>
</tr>
<tr>
<td>ARCH</td>
<td>0.8</td>
<td>4.4</td>
<td>10.8</td>
</tr>
<tr>
<td></td>
<td>λ = 0.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We draw randomly 250 portfolios of 50 stocks for both the ARCH (stocks for which the LM test based on the residuals of the market model is significant at the 5% level) and the No ARCH (LM test based on the residuals of the market model not significant at the 5% level) sub-samples. The daily stock returns series consists of 260 observations.

The following model is estimated:

\[ r_t = \alpha + b r_{\alpha} + \Gamma \delta_{[251;260]} + \epsilon_t, \epsilon_t \sim N(0, h_t) \]

\[ h_t = a_0 + a_1 \epsilon_{t-1}^2 + a_2 h_{t-1} \]

where \( \Gamma \) is the cumulated abnormal returns over the period \([251;260]\) surrounding the event, \( \delta_{[251;260]} = 0.10 \) when \( t \in [251;260] \) and 0 otherwise.

We report the specification of three test statistics:

- J adjusted t-stat
- BMP adjusted t-stat
- Generalized sign test

when the unconditional variance experiments no change, 50%, 200% increase and a 50% decrease.
We draw randomly 250 portfolios of 50 stocks for both the ARCH (stocks for which the LM test based on the residuals of the market model is significant at the 5% level) and the No ARCH (LM test based on the residuals of the market model not significant at the 5% level) sub-samples. The daily stock returns series consists of 260 observations. The series have 260 observations and a 0.2% daily abnormal returns is added for each day in the window (2% cumulated abnormal return).

The following model is estimated:

\[ r_t = \alpha + b r_{mt} + \Gamma_1 \delta \left[t_{[T,t]} \right] + \epsilon_t \]

\[ \epsilon_t \left| \epsilon_{t-1}, \epsilon_{t-2}, \ldots \right. \sim N(0, h_t) \]

\[ h_t = e_{t0} + a_1 \epsilon^2_{t-1} + a_2 h_{t-1} \]

We report the specification of three test statistics:

- J adjusted t-stat
- BMP adjusted t-stat
- Generalized sign test

when the unconditional variance experiments no change, 50%, 200% increase and a 50% decrease.
Table 9: Specification and Power of the Tests Based on Abnormal Returns with NYSE-AMEX data

| Panel A : Specification of the tests | | |
|-------------------------------------|---|---|---|---|---|---|
|                                    | 1% | 5% | 10% | 1% | 5% | 10% |
| J test                             |     |     |     |     |     |     |
| No ARCH                            | 0.8 | 5.6 | 11.2 | 1.2 | 4.4 | 11.2 |
| ARCH                               | 0.4 | 4.8 | 8.4  | 0.4 | 2.4 | 7.2  |
| BMP test                           |     |     |     |     |     |     |
| No ARCH                            | 0.4 | 4.0 | 12.0 | 0.4 | 4.4 | 9.6  |
| ARCH                               | 0.4 | 2.4 | 7.2  | 0.4 | 2.8 | 6.4  |
| G-sign test                        |     |     |     |     |     |     |
| No ARCH                            | 1.2 | 6.0 | 10.4 | 1.6 | 5.6 | 8.4  |
| ARCH                               | 1.2 | 6.0 | 10.4 | 0.8 | 6.0 | 9.2  |

<table>
<thead>
<tr>
<th>Panel B : Power of the tests</th>
</tr>
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<tbody>
<tr>
<td>--------------------------------</td>
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<tr>
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<tr>
<td>J test</td>
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<tr>
<td>No ARCH</td>
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<tr>
<td>ARCH</td>
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<td>BMP test</td>
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<td>ARCH</td>
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<tr>
<td>G-sign test</td>
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<tr>
<td>No ARCH</td>
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<tr>
<td>ARCH</td>
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</tbody>
</table>

We draw randomly 250 portfolios of 50 stocks for both the ARCH (stocks for which the LM test based on the residuals of the market model is significant at the 5% level) and the No ARCH (LM test based on the residuals of the market model not significant at the 5% level) sub-samples, where $y_i$ is the daily abnormal return of stock $i$ on day $k$ over the period $[251;260]$ surrounding the event, $\delta_{ik} = 1$ when $t \in [T_i^1; T_i^2]$ and 0 otherwise. The series have 260 observations. The following model is estimated:

\[
\begin{align*}
    r_{ik} &= \alpha_i + \beta_i \gamma_{ik} + \sum_{k=1}^{K} \gamma_{ik} \varepsilon_{ik} + \varepsilon_{ik}^2, \\
    \varepsilon_{ik} &\sim N(0, h_{ik})
\end{align*}
\]

\[
    h_{ik} = a_0 + a_1 \varepsilon_{ik-1} + a_2 h_{ik-1}
\]

In Panel B, a 1% abnormal returns is added on the fifth day of the window. We report the specification of three test statistics:

- J adjusted t-stat
- BMP adjusted t-stat
- Generalized sign test

when the unconditional variance experiments no change, 50%, 200% increase and a 50% decrease.