Optimal Portfolio Diversification and Product-Market Interactions

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Preliminary and incomplete, comments very welcome.
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Abstract

Although most shareholders hold diversified portfolios, the corporate finance literature postulates that shareholders maximise firm value, while managers sometimes do not. We argue to the contrary that undiversified managers may care more about firm-level risk and return than about the value of their shareholders’ diversified portfolio. These two objectives may differ in presence of product-market interactions. We derive a financial and product market equilibrium in presence of a large, diversified investor and a large number of small shareholders. Stock prices, asset allocation and product-market competition all depend on investors’ risk-aversion, initial endowment, and industry characteristics. We discuss implications to institutional investor activism, executive compensation contracts, venture capital, and the decision to go public.
1 Introduction

Following one of the major recommendations of modern investment theory, investors have held increasingly diversified portfolios, typically through institutional investors. In particular, they routinely hold shares of companies that interact on their product market. For instance, Hansen and Lott (1996) show that in January 1995, nearly 80% of Intel’s shares were held by organizations holding shares in at least one firm among Apple, Compaq, IBM, Microsoft, and Motorola. Yet, the corporate finance literature still widely relies on the idea that shareholders’ objective is to maximise firm value, without much analysis of any potential conflict between firm value maximisation and portfolio value maximisation.¹

In this paper, we argue that the objectives of an undiversified manager may well be closer to firm value maximisation (for a given risk level), while shareholders may instead seek to maximise the value of their diversified portfolio. In presence of product market interactions, this implies that the manager will be tempted to compete more aggressively than the level advocated by his shareholders. We develop this idea by analyzing a financial market and product-market equilibrium in presence of a large, diversified institutional investor and a large number of small shareholders.

Specifically, we develop a model in which an institutional investor mitigates the conflict between undiversified managers and the diversified shareholders. The institutional investor owns shares in two firms. He exerts an effort to influence each firm in order to maximise the value of his portfolio. Meanwhile, he leads the firm to compete less aggressively than if they were owned by different shareholders. This implies that the portfolio of institutional investors is important to individual shareholders because it conveys the information that firms in his portfolio are less likely to compete aggressively. This also implies that stock prices are affected by the institutional investor portfolio holdings. We further show that changes in shareholders ownership in one firm affect the other firm because of both risk-sharing and product market interactions.

¹See, among many others, Markowitz (1952), Sharpe (1964), Murphy (1998), and Grinblatt and Titman (2001)
A number of existing papers emphasize the role of large minority shareholder monitoring. These papers focus on the trade off between the benefits of control, on the one hand, and costs associated with restricted liquidity (Bolton and von Thadden, 1998, Maug, 1998), or adverse managerial incentives (Burkart, Gromb, and Panunzi, 1997, 2000), on the other hand. As in our paper, free-riding from small investors restricts the benefits of trading in most of these papers. However, our paper differs from theirs in that these authors do not address the difference between large diversified and large undiversified investors. While we show that large diversified shareholders who have a portfolio not too different from small investors would tend to reduce the conflict between managers and small investors, a non-diversified large shareholder tends to increase the conflict and encourage managers to compete aggressively at the expense of the small diversified investors. Consequently, none of these papers addresses the portfolio problem of the large monitoring investor. These papers also do not attempt to connect portfolio decisions and product-market competition.

More closely related are Admati, Pfleiderer and Zechner (1994) and DeMarzo and Urosevic (2001). Both of these papers deal with the portfolio decision and monitoring of a large investor. In Admati, Pfleiderer and Zechner (1994), the large shareholder portfolio allocation is determined by risk-sharing motives, while in DeMarzo and Urosevic (2001), the dynamics of trading affects monitoring and portfolio decisions. However, these papers do no address the interactions between the large investor’s portfolio holdings, product-market competition, and the conflicts with other small shareholders. They do not examine either the impact that these have on stock prices and small investors’ portfolio decisions. We are not aware of any work on portfolio decisions and product market competition in an equilibrium framework.

Our results lead to a number of implications. First, when the product-market interactions considered in our model are important, executive compensation contracts should have a greater share of fixed salary and/or industry index, and a lower amount of stocks and stock-options, than has been previously argued in the executive compensation literature, which seems broadly consistent with empirical evidence (Murphy, 1998). In particular, this should be the case more in concentrated, imperfectly competitive industries, than in more competi-
tive industries. In addition, contracts based on relative performance would make diversified investors worse off because it encourages managers to compete aggressively. Again, this is consistent with the lack of popularity of compensation programs with relative performance measure.\(^2\)

The remainder of the paper proceeds as follows. We describe our setting in Section 2. We solve for the financial and product-market equilibrium in section 3. Section 4 develops a number of applications and extensions to venture capital investment, incentives to go public, and the pricing of idiosyncratic risk. Section 5 concludes. The appendix (Section 6) contains all proofs.

### 2 The Model

One institutional investor, \(I\), and a large number of atomistic investors can invest in two firms and a perfectly elastic supply of a riskless asset that generates a rate of return \(r_f = 0\). Investors are endowed with shares in both firms. We term this percentage (of firm shares) \(\alpha_{1\omega}, \alpha_{2\omega}\) in the two firms. All investors in the economy have a CARA utility function. The large investor’s risk aversion coefficient is \(\gamma_I\), and the aggregate risk aversion parameter of the small atomistic investors is \(\gamma_s\).

Institutional investor \(I\) can exert monitoring effort \(E_f, f \in \{1, 2\}, E_f \in \{0, 1\}\) at cost \((1/2)E_f^2\) in order to increase the value of firm’s \(f\) shares. The firms are related in that effort \(E_f\) increases firm \(f\)’s expected return by \(s_fE_f\), but it decreases firm –\(f\)’s expected cashflow by \(c_fE_f\), where \(s_f, c_f\) are real numbers. Unless otherwise specified, we assume that both \(s_f\) and \(c_f\) are positive.\(^3\) The payoff generated by firm \(f\) is denoted by \(r_f \sim (\mu_f, \sigma^2)\), where \(\mu_f = s_fE_f - c_{-f}E_{-f}\) stands for the expected cashflow, and \(\sigma^2\) denotes the variance. For

\(^2\)In the traditional agency literature (Hart and Holmstrom, 1987, Bolton and Dewatripont, 2003), a risk-neutral principal, say an investor, that is concerned about firm-value maximization faces a trade-off between inducing a risk-averse manager to increase firm value and the cost of having him bear risk. Our framework differs from this approach in that we recognize that investors’ objective to maximize the value of their diversified portfolio, while in the absence of an appropriate compensation contract the less diversified manager may be tempted to maximize firm value for a given level of risk.

\(^3\)It should be noted, however, that \(c_f\) can be interpreted as positive spillovers on other firms.
simplicity, we assume that the covariance between the two payoffs is zero. Note that effort increases the total cashflow of the two firms only if $s_f > c_f$. The assumption that firm $f$ has a set of $(s_f, c_f)$ reflects the idea that unless a firm is in a non competitive environment, its projects are likely to hurt competitors. A project that has a high competitive component $c_f$ may even have a negative $s_f - c_f$, which means that an increase in effort $E_f$ results in a decrease in the total cashflow of the two firms due to a high competitive component and high deadweight costs, e.g. costs associated with advertising.\footnote{4} Hence, a fraction of $s_f$ comes from the increase in the opportunity set and some comes from competitive behavior that hurt its competitor.\footnote{5}

The timing is as follows:

At $t = 0$, investors choose their holdings. The institutional investor holds $\alpha_1, \alpha_2$ of the shares in firm 1, 2 respectively. Small investors hold $1 - \alpha_1, 1 - \alpha_2$ in firm 1,2 respectively. These holdings are reached through trading at market price.

At $t = 1$, given the equilibrium allocation at $t = 0$, $I$ chooses effort invested in the two firms $E_1, E_2$.

At $t = 2$, the cashflows are generated and the payoffs distributed to investors.

\section{Financial Market Equilibrium}

We proceed by backward induction.

\footnote{4}Some institutional investors are often viewed as rather passive investors, that at most approve or disapprove management actions. If one assumes that $I$'s monitoring technology is of this type, a more natural but equivalent interpretation of effort is $M_f = 1 - E_f$, $M_f \in \{0, 1\}$. Under this interpretation, the expected payoff of firm $f$ is decreasing in monitoring, because monitoring reduces firm's $f$ managers ability to take on competitive projects. Thus, an increase in $M_f$, and a decrease in $E_f$, have the same effect: They both reduce firm's $f$'s cash flow and they both increase firm $-f$ cashflow.

\footnote{5}In our setting, an increase in effort leads to more competition. Yet, the combined value of the two firms is typically maximized for levels of effort that are lower than the effort levels that would maximize the value of the firms taken separately. This is, of course, because we focus on the value of the firms, and we abstract from the deadweight costs associated with a decrease in competition. Such costs may be captured by considering a third risky asset that has an expected return of $d_1E_1 + d_2E_2$, with $d_1 > c_1$ and $d_2 > c_2$. This third asset may represent consumer surplus or labor income as well as other, non-traded firms. We develop this idea in Section 5.
At time $t = 1$, $I$ chooses effort so as to solve

$$
\max_{E_1, E_2} U(E_1, E_2) = \alpha_1 \mu_1 + \alpha_2 \mu_2 - \frac{\gamma_s}{2} \sigma^2 (\alpha_1^2 + \alpha_2^2) - \frac{E_1^2}{2} - \frac{E_2^2}{2}.
$$

(1)

The first two terms reflect the expected payoff from holding the portfolio of the two firms. The third term reflects the cost of risk of having such a portfolio. The last two terms are $I$’s monitoring cost.

This maximization problem implies that effort satisfies the following first-order conditions

$$
E_1 = (\alpha_1 s_1 - \alpha_2 c_1) \quad (2)
$$

$$
E_2 = (\alpha_2 s_2 - \alpha_1 c_2)
$$

Hence, effort in firm $f$ increases with the fraction of shares that $I$ holds in firm $f$, and decreases with the fraction of shares that he holds in firm $-f$. In addition, effort $E_f$ increases with $s_f$ and decreases with $c_f$.

A measure of competition is the degree of effort invested in this economy, hence, $E_1 + E_2$.

$$
\Phi = E_1 + E_2 = (s_1 - c_2) \alpha_1 + (s_2 - c_1) \alpha_2 \quad (3)
$$

At time $t = 0$, small investors maximize utility and choose the portion of investment in firm 1 and 2. We define their initial endowments $\beta_1, \beta_2$ and their final holdings $\beta_1, \beta_2$. The small investors conjecture that $I$’s final holdings are $\alpha_1$ and $\alpha_2$ so they can derive the expected payoff of both firms. Formally, the small investors maximize,

$$
\max_{\beta_1, \beta_2} U(\beta_1, \beta_2) = \beta_1 \mu_1 + \beta_2 \mu_2 - (\beta_1 - \beta_1 \omega) p_1 - (\beta_2 - \beta_2 \omega) p_2 - \frac{\gamma_s}{2} \sigma^2 (\beta_1^2 + \beta_2^2)
$$

(4)

The first three terms represent the expected payoff from having a portfolio of two risky assets and a riskless asset. The forth term is the cost of bearing risk. This problem is solved for

$$
p_1 = \mu_1 - \gamma_s \sigma^2 \beta_1 \quad (5)
$$

$$
p_2 = \mu_2 - \gamma_s \sigma^2 \beta_2
$$
Firm $f$’s stock price increases in its expected cashflow and decreases in small investors’ risk-aversion and portfolio holdings. Since the market-clearing condition requires $\beta_1 = 1 - \alpha_1$, $\beta_2 = 1 - \alpha_2$ prices solve for

\[
p_1 = \mu_1 - \gamma_s \sigma^2 (1 - \alpha_1) \\
= s_1 (\alpha_1 s_1 - \alpha_2 c_1) - c_2 (\alpha_2 s_2 - \alpha_1 c_2) - \gamma_s \sigma^2 (1 - \alpha_1) \\
= (s_1^2 + c_2^2) \alpha_1 - (s_1 c_1 + c_2 s_2) \alpha_2 - \gamma_s \sigma^2 (1 - \alpha_1)
\]

\[
p_2 = \mu_2 - \gamma_s \sigma^2 (1 - \alpha_2) \\
= (s_2^2 + c_1^2) \alpha_2 - (s_1 c_1 + c_2 s_2) \alpha_1 - \gamma_s \sigma^2 (1 - \alpha_2)
\]

As in Admati et al (1994) and Demarzo and Urosevic (2001), we assume that $I$ acts strategically and maximizes utility given its effect on prices. Thus, $I$ is not a price taker, and takes into account the effect of his holdings on the equilibrium price.

\[
\max_{\alpha_1, \alpha_2} U(\alpha_1, \alpha_2) = \alpha_1 \mu_1 + \alpha_2 \mu_2 - (\alpha_1 - \alpha_{1.1}) p_1 - (\alpha_2 - \alpha_{2.2}) p_2 - \frac{\gamma_I}{2} \sigma^2 (\alpha_1^2 + \alpha_2^2) - \frac{E_1^2}{2} - \frac{E_2^2}{2} 
\]

As in the small investors case, the first three terms is the expected payoff of the portfolio and the forth term is the cost of risk. However, $I$ also accounts for the cost of monitoring.

**Proposition 1** In equilibrium, portfolio allocations, stock prices and efforts satisfy

\[
\alpha_1 = \frac{\alpha_{1.1}(B_1 D_2 - A^2) + \alpha_{2.2} A(B_2 - D_2) + AC + CD_2}{D_1 D_2 - A^2} \\
\alpha_2 = \frac{\alpha_{2.2}(B_2 D_1 - A^2) + \alpha_{1.1} A(B_1 - D_1) + AC + CD_1}{D_1 D_2 - A^2} \\
p_1 = \max [0, B_1 \alpha_1 - A \alpha_2 - C], \quad p_2 = \max [0, B_2 \alpha_2 - A \alpha_1 - C] \\
E_1 = [0, \alpha_1 s_1 - \alpha_2 c_1], \quad E_2 = \max [0, \alpha_2 s_2 - \alpha_1 c_2]
\]

where,

\[
A = (s_1 c_1 + c_2 s_2)
\]
We have to discuss negative $\alpha$, $\alpha > 1$, negative prices, etc.

We now discuss the intuitions behind these results. We consider the benchmark cases of no competition and pure competition before going back to our full model.

### 3.1 Case 1: No competition

We now illustrate the case where firms do not compete by considering the case where $c_1 = c_2 = 0$. An increase in effort $E_1$ increases the cash flow of firm 1, but it does not reduce the cash flow of firm 2. One interpretation is that $I$ can costlessly distinguish projects that are competitive from projects that do not, and therefore he exerts effort that only increases the opportunity set. Without loss of generality, we restrict the discussion to firm 1. Substituting $c_1 = c_2 = 0$ into the equilibrium holdings we derive, the equilibrium $\alpha_1$ and $p_1$ simplify to

$$\begin{align*}
\alpha_1 &= \frac{\alpha_{1\omega} (s_1^2 + \gamma_s \sigma^2) + \gamma_s \sigma^2}{2\gamma_s \sigma^2 + \gamma_I \sigma^2 + s_1^2} \\

p_1 &= \left( s_1^2 + \gamma_s \sigma^2 \right) \frac{\alpha_{1\omega} (s_1^2 + \gamma_s \sigma^2) + \gamma_s \sigma^2}{2\gamma_s \sigma^2 + \gamma_I \sigma^2 + s_1^2} - \gamma_s \sigma^2
\end{align*}$$

**Proposition 2** The institutional investor’s holding in firm 1, $\alpha_1$, increases with $\alpha_{1\omega}$, and $\gamma_s$; it decreases with $\gamma_I$; it is independent of $\alpha_{2\omega}$ and $s_2$; it increases in $s_1$ (resp. decreases in $\sigma^2$) if $\alpha_{1\omega} > \frac{\gamma_s}{(\gamma_s + \gamma_I)}$ and it decreases in $s_1$ (resp. increases in $\sigma^2$) otherwise.

Firm 1’s stock price, $p_1$, increases with $\alpha_{1\omega}$ and $s_1$; it decreases in $\gamma_I$; it is independent of $\alpha_{2\omega}$ and $s_2$; it may either increase or decrease in $\gamma_s$ and in $\sigma^2$.

When $\alpha_{1\omega}$ is higher, it pays off for $I$ to exert more effort. Since small shareholders free-ride on the effort made by $I$, the institutional investors gains primarily on the shares that he owned initially. Hence, when $I$’s initial holdings in firm 1 increase, he exerts more effort, and the gain on his initial endowment $\alpha_{1\omega}$ is higher. When the risk aversion of the small
aggregate investors is high, the market prices risk more heavily, and $I$ is tempted to hold more shares in firm 1 and bear the risk until it is realized. On the other hand, an increase in $\gamma_I$ increases $I$’s incentive to sell the shares, because his cost of bearing the risk becomes higher relative to the market price. In this framework it does not come at a surprise that changes in $s_2$ and $\alpha_{2,\omega}$ do no affect $I$ holding of $\alpha_1$ since firms do not interact on the product market\(^6\).

An increase in $s_1$ can either increase or reduce $\alpha_1$, depending on the risk aversion coefficients, and on endowment $\alpha_{1,\omega}$. If the market has a relatively high risk aversion compared to $I$, the increase in price $p_1$ due to the increase in $s_1$ might induce $I$ to sell more shares if his initial endowment $\alpha_{1,\omega}$ is low.

Interestingly, $\alpha_1$ is not necessarily decreasing with respect to the variance. The derivative decreases with $\alpha_{1,\omega}$ and $\gamma_I$ and increases with $\gamma_s$. Thus, the higher $I$’s initial endowment of $I$ and the more risk averse he is, the more he tends to reduce his holding in $\alpha_1$. On the other hand, if $\gamma_s$ is relatively high, $I$ may increase $\alpha_1$ because the market prices risk heavily and this effect may dominate the fact that $I$’s initial holdings are riskier.

As for $\alpha_1$, firm 1’s stock price increases in $\alpha_{1,\omega}$. This is a direct result of the increase in $\alpha_1$ which induces $I$ to exert a higher effort level at $t = 1$. There are two reasons for the stock price to decrease in $\gamma_I$: (1) $I$ tends to supply the market with more shares due to his higher risk aversion; and (2) the smaller amount of shares makes $I$ exert less effort, and this reduces the firm’s expected cash flow.

The derivative $\frac{\partial p_1}{\partial s}$ can be either positive or negative. When $\gamma_s$ increases, the market prices risk more, which induces $I$ to increase his holdings $\alpha_1$, and wait for the realization of the risk in period $t = 2$. Hence, $\gamma_s$ has two effects: (1) Risk is priced more heavily resulting in an initial decrease in price $p_1$, (2) $I$ increases $\alpha_1$, which results in more effort $E_1$ and a higher $\mu_1$. If $\alpha_{1,\omega} < 0.5$ the first effect always dominates and the result is a lower price. For very high $\alpha_{1,\omega}$ the second effect can dominate because $I$ holds most of the supply of shares.

\(^6\)The assumption that there is zero correlation between the two stocks is also a determinant of the irrelevance of these two variables. This simplifying assumption enables us to focus on the role of product-market interactions.
in the market.

Price $p_1$ increases in $s_1$. Thus, even if $I$ reduces his holding $\alpha_1$ and, therefore, reduces effort $E_1$, it is a second order effect compared to the initial price increase effect of the higher $s_1$. Similar to the effect on $\alpha_1$, the derivative of $p_1$ with respect to $\sigma^2$ is also ambiguous. In general, an increase in volatility decreases stock prices. However, this depends to a large degree on $\alpha_1$, $\gamma_1$, $\gamma_s$. The derivative decreases with $\alpha_1$ and $\gamma_1$ and increases with $\gamma_s$. Thus, the more is the initial endowment of $I$ of firm 1 shares and the more he is risk averse, the more he tends to reduce his holding in $\alpha_1$ which reduces price due to the increased supply and less $t = 1$ effort. On the other hand, if $\gamma_s$ is relatively high, $I$ prefers to increase $\alpha_1$ because the increased variance is priced heavily in the market. This increase in $\alpha_1$ may dominate and result in an increase in market price.

### 3.2 Case 2: Pure competition

In this case, the effort exerted by $I$ in firm 1 increases firm 1’s expected cash flow and reduces firm 2’s expected cash flow by the same amount: $s_1 = c_1$, $s_2 = c_2$. In other words, we assume that effort does not increase the opportunity set.

Substituting $s_1 = c_1$ and $s_2 = c_2$ in firm 1’s stock price, we obtain

\[
E_1 = s_1 (\alpha_1 - \alpha_2), \quad E_2 = s_2 (\alpha_2 - \alpha_1) \\
p_1 = (c_1^2 + c_2^2) (\alpha_1 - \alpha_2) - \gamma_s \sigma^2 (1 - \alpha_1)
\]

(10)

Note that if $\alpha_2 > \alpha_1$, in the absence of non-negativity restrictions, effort $E_1$ and price $p_1$ would be negative. The intuition here is that when $I$ holds more shares in firm 2 than in firm 1, he exerts effort only in firm 2 because the value created by the effort invested in firm 1 completely offsets the value it destroys in firm 2. Since effort is costly, a unit of effort in firm 1 would decrease firm 2’s expected return by the same amount as it increases firm 1’s expected return. This would make $I$ worse off, because his holdings in firm 2 are larger. Since reducing $\alpha_1$ decreases the cost of exerting effort in firm 2, the equilibrium firm 1’s holding is $\alpha_1 = 0$. Since $I$ does not hold shares in firm 1 and since he does not exert effort in firm 1, firm 1 will generate a negative return, i.e. $\mu_1 = -c_2 E_2$. Small investors do not want
to hold firm 1 shares either, because they generate a negative cash flow with a risk of $\sigma^2$, making the riskless asset dominate an investment in firm 1. Firm 1 would then exit, and only firm 2 remains in the market. Ultimately, the market compares investing only in firm 1 or only in firm 2, and the firm that survives is the one that has the higher productivity. Thus, pure competition implies that only one firm remains, i.e. we have a monopolistic outcome.

The effects described above also hold if the total gain from competitive behavior is negative, i.e., if $s_1 < c_1$. This scenario reflects the idea that a firm is attractive to investors only if they do not invest in the other firm. However, investors are then better off investing in both firms and preventing them from competing. But this investment is again dominated by an investment in the better firm and riskless asset. In our setting, a firm must invest in projects that increase the opportunity set in order to remain on the market for sure. A strategy that only competes with other firms, will eventually drive the firm out of business. Since only one firm remains, the portfolio decision is reduced to an allocation between the unique risky asset, firm 1’s shares, and the riskless asset, and all comparative statics results are identical to the case of two firms with no interaction.\textsuperscript{7}

\section*{3.3 Case 3: The full model}

From the previous section, the existence of both firms requires $s_1 > c_1$, and $s_2 > c_2$. When $c_1, c_2$ are different from 0, product market competition affects portfolio allocations and stock prices as follows

\textbf{Proposition 3} Portfolio allocation in firm $f$, $\alpha_f$, increase in $I$’s initial allocation in firm $f$, decrease in initial allocation in firm -$f$. Firm $f$’s stock price increase in $I$’s initial allocation in firm $f$,

\textbf{Intuitions} \footnote{This is due, of course, to our assumption that $I$ must exert effort in a firm for its expected return to be positive. One possible interpretation is that effort in a firm destroys the other firm’s purpose by making its technology unattractive to consumers. There is an analogy with pure price competition with entry costs, where a second firm never wants to bear the entry cost and then make zero profit on the output market.}
4 Applications and Extensions

4.1 Venture Capital Investment

Venture capitalists hold portfolios of shares in a wide range of firms. Our large shareholder may be such a venture capitalist. It may be the case that this venture capitalist is asked to finance two competing projects. The VC may be tempted to invest in both projects for risk-sharing purposes. Even if he does so, he may be tempted to reduce competition between both projects. For instance, if one project has only one possible application, e.g. a superior hair implant technique, and the other project may develop into either another hair implant technique or a hair removal technique, the VC will be tempted to encourage the second firm to favor the hair removal technique, even though this reduces the value of the second firm.

Our analysis further implies that when negotiating the terms of VC financing, the terms of the VC contract will be affected by the VC’s portfolio. When the firm is taken public at a later stage, the IPO price should also depend on the VC’s portfolio\(^8\).

4.2 The Invisible Firm

4.2.1 Private companies

The model can be extended to account for a third firm whose shares are not traded. This enables us to derive some empirical predictions.

Consider three firms with the following expected returns,

\[
\begin{align*}
\mu_1 &= s_1E_1 - c_{12}E_2 - c_{13} \\
\mu_2 &= -c_{21}E_1 + s_2E_2 - c_{23} \\
\mu_3 &= -c_{31}E_1 - c_{32}E_2 + s_3
\end{align*}
\]

(11)

where as before \(E_1, E_2\) is the effort exerted by institutional investor \(I\), \(s_i E_i\) is the increase in expected payoff in firm \(i\) due to effort \(E_i\), and \(c_{ij} E_j\) is the reduction in expected payoff in

\(^8\)This argument is particularly important to corporate venture capitalists who finance a project that is related to their business activities on the product market.
firm $i$ due to the effort that is invested in firm $j$. We assume that firm 3 is not traded and can not be held by $I$. This could be the case if $I$ is for example, a mutual fund which must hold only traded assets. Thus, for $I$ firm 3 is an invisible firm and he can not trade in firm 3 shares, nor can he exert effort to affect its payoff. In broad terms, all decisions in firm 3 are made by either the manager or the large individual non-diversified shareholders. Hence $I$ can not effect the competitive behavior of firm 3, and the effect of firm 3 on $\mu_1$ and $\mu_2$ (which is $c_{13}, c_{23}$ respectively) is exogenous to $I$. This means that $I$ does not internalizes the fact that he competes with the private firm 3, if he invests more effort in the public firms 1 and 2. The result of course is that firm 3 is hurt by the fact that $I$ is fine with a aggressive behavior of firm 1 and 2, as long as it generally hurts firm 3 (as long as $c_{21}E_1$ and $c_{12}E_2$ are relatively small to $c_{31}$ and $c_{32}$).

The result above enables us to achieve another motive for firms to go public. When a private firm goes public it joins the set of firms that are held by diversified shareholders. By joining this set, the firm is less subject to competitive behavior from other public firms in the industry. While managers of public firms can compete with private firms easily, it is harder to compete with other public companies, because the diversified shareholders would oppose it.\(^9\) This allows us to derive the prediction that as more firms in the industry become public, the more it would benefit other firms in the industry to become public. Thus, IPO would tend to follow a herding behavior of firms from the same industry. Finally, we provide a rational for the empirical observations of IPO rationing. The evidence to date suggests that where book building is used, institutional investors receive preferential allocations. Using U.S. data, Aggarwal, Prabhala, and Puri (2002) and Hanley and Wilhem (1995) find that institutions are favored, as do Cornelli and Goldreich (2001) with British data. If underwriters are concerned with market stability, issuing IPOs to institutional investor is a commitment device against competition from other public firms. The result is that even if the new IPO

\(^9\)Note however, that it is not enough for the firm to want to go public. To go public, the firm must have an ability to increase the opportunity set of the economy, i.e., $s_f > c_f$. If the firm does not have this capability, $I$ wont invest in the firm, and diversified shareholders would price the shares at zero since the firm would still be subject to competition from other firms.
enables the firm to raise much capital in the financial markets, it will not enable the firm to use this capital against other public firms in the industry. The institutional investor, who hold shares in those other firms (potentially subject to competition), would oppose such competitive strategy. This means that to avoid reduction in market price of other firms, the best way is to ration the IPO shares to institutional investors.

Furthermore, we show that one aspect of the IPO decision may be to provide ownership by large shareholders that also own shares in competing firms. By becoming a public firm that is held by a large diversified investor, the company enters a “safe haven” from competitive pressure of other firms in the portfolio of the large diversified shareholder. This means that a firm’s decision of going public depends to a large extent on whether its competitors are private or public. This allows us to derive the prediction that as more firms in the industry become public, the more it would benefit other firms in the industry to become public. Thus, IPO would tend to follow a herding behavior of firms from the same industry. We also provide an explanation to the empirical fact of share rationing to institutional investors during IPOs (Aggarwal et al, 2002, Hanley and Wilhem, 1995, Cornelli and Goldreich, 2002). To avoid a decrease in prices of other firms in the industry, underwriters prefer to give the shares to institutional investors. This kind of rationing serves as a commitment device that other public firms in the economy are not going to be hurt by the new IPO company, and the result is that the market value of the public firms is not reduced by the new IPO.

4.2.2 Public Policy

In our setting, the combined value of the two firms is maximized for levels of effort that are lower than the effort levels that would maximize the value of the firms taken separately. This leads to deadweight costs associated with a decrease in competition. Such costs may be captured by considering a third non-traded risky asset that represents consumer surplus and has an expected return of \( d_1 E_1 + d_2 E_2 \), with \( d_1 > c_1 \) and \( d_2 > c_2 \). While, investors care about firm profitability of their portfolio, and therefore reduce effort due to competitive behavior, consumers are concerned with consumer surplus and want firms to increase effort
as much as possible. Thus, the fact that investors are diversified may hurt consumer surplus. In general, if \( I \) is a mutual fund, the SEC regulations prohibits it from have a non-diversified portfolio. The Investment Company Act does not allow a fund to hold more than 5% of its assets in any firm’s security. It also does not allow a fund to hold more than 5% of any issue. This makes mutual funds diversified, and according to our model, hurts the competition in the market. If it was up to the Bureau of Competition to regulate mutual funds, it would advance an agenda that prohibits mutual funds to diversify and not vise versa. Note the difference between these two agencies. The SEC defines its primary mission as protecting investors, while the Federal Trade Commission defines its mission as working for consumer protection and a competitive market place.

### 4.3 Capital budgeting

The conflict between diversified and non-diversified investors is in the way capital budgeting decisions can affect the way in which capital budgeting decisions are made. In the standard finance textbook, a project’s NPV is estimated by calculating the expected cash flow for the firm and discounting it by the weighted average cost of capital (WACC). The WACC is derived by estimating the expected rate of return demanded by investors, which is usually done by asset pricing model such as the CAPM. Thus, the expected cash flow is only that of the firm, while the WACC is calculated under the assumption that investors are diversified and therefore require a higher expected return only for non-diversifiable risk.

This begs the question, if investors are diversified, why look only at the expected cash flow of the firm and not all the firms in the economy that are affected by the project? If investors are completely diversified then they care about all the firms in their portfolio. Moreover, since we are usually assuming a complete market and a CAPM world, expected cashflow should be calculated for all the firms in the economy since all investors hold the market. The usual reasoning for this inconsistency is since we also assume a competitive market, each firm assumes that it has no influence on other firms’ cash flow. However, in a different framework, Rubinstein (1978, 2002) shows that even the assumption that in a competitive market firms
have no affect on the interest rate is faulty. Making an analogy to his argument, even if managers assume that they do not affect the cash flow of other firms, diversified investors who have a very small share in each firm, cannot make the same assumption. Rubinstein (1978, 2002) shows that the difference between presuming an arbitrarily small influence and zero influence will dramatically affect the unanimity conclusion. From the point of view of diversified investors, even if each firm has a very small impact on the cash flow of other firms, this small effect dominates the value maximization criterion of any specific firm, which also comprises only a small portion of the consumer's portfolio.

Thus, the numerator of the NPV calculation is the expected cash flow of the firm, which would be advanced by undiversified managers but not diversified investors who care about expected cash flow in all the firms in their portfolio. The denominator of the NPV calculation is the discount rate demanded by diversified investors but not that demanded by undiversified managers who are concerned only with firm value.

5 Conclusion

This paper has considered interactions between portfolio allocation and product-market strategy. The fact that imperfect competition affects portfolio allocations and stock prices suggests the price of firm-specific risk may be positive. This may lead to a model of asset pricing that is based on product-market characteristics. The interactions between industrial organization, corporate finance and asset pricing may well prove a promising avenue for future research.
6 Appendix

6.1 Proof of Proposition 1

\( U(\alpha_1, \alpha_2) \) can be rewritten

\[
U(\alpha_1, \alpha_2) = \frac{E_1^2}{2} + \frac{E_2^2}{2} - \frac{\gamma_I}{2} \sigma^2 (\alpha_1^2 + \alpha_2^2) - (\alpha_1 - \alpha_{1\omega}) p_1 - (\alpha_2 - \alpha_{2\omega}) p_2 \\
= \frac{(\alpha_1 s_1 - \alpha_2 c_1)^2}{2} + \frac{(\alpha_2 s_2 - \alpha_1 c_2)^2}{2} - \frac{\gamma_I}{2} \sigma^2 (\alpha_1^2 + \alpha_2^2) \\
- (\alpha_1 - \alpha_{1\omega}) ((s_1^2 + c_2^2) \alpha_1 - (s_1 c_1 + c_2 s_2) \alpha_2 - \gamma_s \sigma^2 (1 - \alpha_1)) \\
- (\alpha_2 - \alpha_{2\omega}) ((s_2^2 + c_1^2) \alpha_2 - (s_1 c_1 + c_2 s_2) \alpha_1 - \gamma_s \sigma^2 (1 - \alpha_2))
\]

The first order conditions can be written

\[
\frac{\partial U(\alpha_1, \alpha_2)}{\partial \alpha_1} = - (s_1^2 + c_2^2 + (\gamma_I + 2\gamma_s) \sigma^2) \alpha_1 + (s_1 c_1 + c_2 s_2) (\alpha_2 - \alpha_{2\omega}) \\
+ (s_1^2 + c_2^2 + \gamma_s \sigma^2) \alpha_{1\omega} + \gamma_s \sigma^2 = 0
\]

which gives the reaction function

\[
\alpha_1 = \frac{(\alpha_2 - \alpha_{2\omega}) (s_1 c_1 + c_2 s_2) + \alpha_{1\omega} (s_1^2 + c_2^2 + \gamma_s \sigma^2) + \gamma_s \sigma^2}{(2\gamma_s \sigma^2 + \gamma_I \sigma^2 + s_1^2 + c_2^2)}
\]

and

\[
\frac{\partial U(\alpha_1, \alpha_2)}{\partial \alpha_2} = (s_1 c_1 + c_2 s_2) (\alpha_1 - \alpha_{1\omega}) - (s_2^2 + c_1^2 + \gamma_I \sigma^2 + 2\gamma_s \sigma^2) \alpha_2 \\
+ (s_2^2 + c_1^2 + \gamma_s \sigma^2) \alpha_{2\omega} + \gamma_s \sigma^2 = 0
\]

which allows us to write

\[
\alpha_2 = \frac{(\alpha_1 - \alpha_{1\omega}) (c_1 s_1 + c_2 s_2) + \alpha_{2\omega} (s_2^2 + c_1^2 + \gamma_s \sigma^2) + \gamma_s \sigma^2}{(2\gamma_s \sigma^2 + \gamma_I \sigma^2 + s_2^2 + c_1^2)}
\]
For ease of exposition, we rewrite

\[ \alpha_1 = \frac{A(\alpha_2 - \alpha_{1\omega}) + B_1\alpha_{1\omega} + C}{D_1} \]

\[ \alpha_2 = \frac{A(\alpha_1 - \alpha_{1\omega}) + B_2\alpha_{2\omega} + C}{D_2} \]

where

\[ A = (s_1c_1 + c_2s_2) \]

\[ B_1 = (s_1^2 + c_1^2 + \gamma s \sigma^2) \quad \text{;} \quad B_2 = (s_2^2 + c_2^2 + \gamma s \sigma^2) \]

\[ C = \gamma s \sigma^2 \]

\[ D_1 = (2\gamma s \sigma^2 + \gamma_1 \sigma^2 + s_1^2 + c_2^2); \quad D_2 = (2\gamma s \sigma^2 + \gamma_1 \sigma^2 + s_2^2 + c_1^2) \]

Note that: \( B_1 - D_1 = B_2 - D_2 = -(\gamma s \sigma^2 + \gamma_1 \sigma^2), D_1D_2 - A_2 > 0, B_1D_2 - A^2 > 0, \) and \( B_2D_1 - A^2 > 0. \)

Clearly, these first-order conditions define a unique maximum, as \( \frac{\partial^2 U}{\partial \alpha_1^2} < 0, \)

\( \frac{\partial^2 U}{\partial \alpha_2^2} < 0, \) and \( \Delta = \left( \frac{\partial^2 U}{\partial \alpha_1 \partial \alpha_2} \right)^2 - \left( \frac{\partial^2 U}{\partial \alpha_1^2} \right) \left( \frac{\partial^2 U}{\partial \alpha_2^2} \right) < 0. \)

Solving for \( \alpha_1 \) and \( \alpha_2, \) we obtain

\[ \alpha_1 = \frac{\alpha_{1\omega}(B_1D_2 - A^2) + \alpha_{2\omega}A(B_2 - D_2) + C(A + D_2)}{D_1D_2 - A^2} \]

\[ \alpha_2 = \frac{\alpha_{2\omega}(B_2D_1 - A^2) + \alpha_{1\omega}A(B_1 - D_1) + C(A + D_1)}{D_1D_2 - A^2} \]

Plugging \( \alpha_1 \) and \( \alpha_2 \) in prices, we obtain

\[ p_1 = B_1\alpha_1 - A\alpha_2 - C \]

\[ = \frac{\alpha_{1\omega}[B_1(B_1D_2 - 2A^2) + A^2D_1] + \alpha_{2\omega}A[B_1(B_2 - D_2) - (B_2D_1 - A^2)]}{D_1D_2 - A^2} + \frac{C(A - D_2)(B_1 - D_1)}{D_1D_2 - A^2} \]

\[ p_2 = B_2\alpha_2 - A\alpha_1 - C \]

\[ = \frac{\alpha_{2\omega}[B_2(B_2D_1 - 2A^2) + A^2D_2] + \alpha_{1\omega}A[B_2(B_1 - D_1) - (B_1D_2 - A^2)]}{D_1D_2 - A^2} + C(B_2 - D_2)(A - D_1)}{D_1D_2 - A^2} \]
6.2 Proof of Proposition 2

No competition: \( c_1 = c_2 = 0 \). Thanks to symmetry of our setting, we can, without loss of generality, restrict our discussion to firm 1.

This implies \( A = 0, B_1 = s_1^2 + \gamma_s\sigma^2 ; C = \gamma_s\sigma^2, D_1 = 2\gamma_s\sigma^2 + \gamma_I\sigma^2 + s_1^2 \).

\[
\alpha_1 = \frac{\alpha_{1w}B_1 + C}{D_1} = \frac{\alpha_{1w}(s_1^2 + \gamma_s\sigma^2) + \gamma_s\sigma^2}{2\gamma_s\sigma^2 + \gamma_I\sigma^2 + s_1^2}
\]

Because of the absence of competition, \( \alpha_1 \) is independent of \( s_2 \) and \( \alpha_{1w} \). We now take the partial derivatives of \( \alpha_1 \) with respect to \( \alpha_{1w}, \gamma_I, \gamma_s, s_1, \) and \( \sigma^2 \).

\[
\frac{\partial \alpha_1}{\partial \alpha_{1w}} = \frac{s_1^2 + \gamma_s\sigma^2}{2\gamma_s\sigma^2 + \gamma_I\sigma^2 + s_1^2} > 0
\]

\[
\frac{\partial \alpha_1}{\partial \gamma_I} = -\sigma^2\alpha_{1w}s_1^2 + \alpha_{1w}\gamma_s\sigma^2 + \gamma_s\sigma^2 \left( \frac{2\gamma_s\sigma^2 + \gamma_I\sigma^2 + s_1^2}{(2\gamma_s\sigma^2 + \gamma_I\sigma^2 + s_1^2)^2} \right) < 0
\]

\[
\frac{\partial \alpha_1}{\partial \gamma_s} = \sigma^2\alpha_{1w}\gamma_I\sigma^2 + (1 - \alpha_{1w})s_1^2 + \gamma_I\sigma^2 > 0
\]

\[
\frac{\partial \alpha_1}{\partial s_1} = \frac{2\sigma^2(\gamma_s + \gamma_I)\alpha_{1w} - \gamma_s}{(2\gamma_s\sigma^2 + \gamma_I\sigma^2 + s_1^2)^2}
\]

\[
\frac{\partial \alpha_1}{\partial \sigma^2} = \frac{s_1^2[\gamma_s - \alpha_{1w}(\gamma_s + \gamma_I)]}{(2\gamma_s\sigma^2 + \gamma_I\sigma^2 + s_1^2)^2}
\]

Proceeding similarly for \( p_1 \), we write \( p_1 \) as

\[
p_1 = (s_1^2 + \gamma_s\sigma^2)\alpha_1 - \gamma_s\sigma^2 = s_1^2\alpha_1 - \gamma_s\sigma^2(1 - \alpha_1)
\]

\[
= (s_1^2 + \gamma_s\sigma^2)\alpha_{1w}(s_1^2 + \gamma_s\sigma^2) + \gamma_s\sigma^2 - \gamma_s\sigma^2
\]

\[
= [1 - \frac{(\gamma_s - \gamma_I)\sigma^2}{2\gamma_s\sigma^2 + \gamma_I\sigma^2 + s_1^2}][\alpha_{1w}(s_1^2 + \gamma_s\sigma^2) + \gamma_s\sigma^2] - \gamma_s\sigma^2
\]
\[
\frac{\partial p_1}{\partial \alpha_{1\omega}} = \frac{(s_1^2 + \gamma_s \sigma^2)^2}{2\gamma_s \sigma^2 + \gamma_f \sigma^2 + s_1^2} > 0 \\
\frac{\partial p_1}{\partial \gamma_f} = -\left(s_1^2 + \gamma_s \sigma^2\right) (\alpha_{1\omega} s_1^2 + \alpha_{1\omega} \gamma_s \sigma^2 + \gamma_s \sigma^2) \left(\frac{\sigma^2}{2\gamma_s \sigma^2 + \gamma_f \sigma^2 + s_1^2}\right)^2 < 0 \\
\frac{\partial p_1}{\partial \gamma_s} = \left[1 - \left(\frac{(\gamma_s - \gamma_f) \sigma^2}{2\gamma_s \sigma^2 + \gamma_f \sigma^2 + s_1^2}\right)(1 + \alpha_{1\omega})\gamma_s \sigma^2 \right] - \frac{3\gamma_f \sigma^2 + s_1^2}{(2\gamma_s \sigma^2 + \gamma_f \sigma^2 + s_1^2)^2}\left[\gamma_{1\omega} (s_1^2 + \gamma_s \sigma^2) + \gamma_s \sigma^2\right] - \gamma_s \\
\frac{\partial p_1}{\partial s_1} = 2s_1 [4\alpha_{1\omega} \gamma_s \sigma^2 s_1^2 + 2\alpha_{1\omega} \gamma_f \sigma^2 s_1^2 + \alpha_{1\omega} s_1^4 + 3\alpha_{1\omega} \gamma_s^2 \sigma^2 + 2\alpha_{1\omega} \gamma_s \sigma^4 \gamma_f + \gamma_f \sigma^4 + \gamma_s \sigma^4 \gamma_f] > 0 \\
\frac{\partial p_1}{\partial \sigma^2} = \frac{\gamma_s [2 \gamma_s \sigma^2 + \gamma_f \sigma^2 + s_1^2] \left[\alpha_{1\omega} (s_1^2 + \gamma_s \sigma^2) + (s_1^2 + \gamma_s \sigma^2) (1 + \alpha_{1\omega}) - (\gamma_s + \gamma_f) \sigma^2 - s_1^2\right]}{(2\gamma_s \sigma^2 + \gamma_f \sigma^2 + s_1^2)^2} \\
\frac{\partial p_1}{\partial \sigma^2} - \frac{(s_1^2 + \gamma_s \sigma^2) \left[\alpha_{1\omega} (s_1^2 + \gamma_s \sigma^2) + \gamma_s \sigma^2\right]}{(2\gamma_s \sigma^2 + \gamma_f \sigma^2 + s_1^2)^2} \\

6.3 \hspace{1em} \text{Proof of Proposition 3}

We rewrite portfolio allocations and prices as

\[
\alpha_1 = \frac{\alpha_{1\omega} H_1 + \alpha_{2\omega} AF + C(A + D_2)}{G} \\
\alpha_2 = \frac{\alpha_{2\omega} H_2 + \alpha_{1\omega} AF + C(A + D_1)}{G} \\
p_1 = \left(s_1^2 + c_2^2 + \gamma_s \sigma^2\right) \alpha_1 - \left(s_1 c_1 + c_2 s_2\right) \alpha_2 - \gamma_s \sigma^2 \\
p_2 = \left(s_2^2 + c_1^2 + \gamma_s \sigma^2\right) \alpha_2 - \left(s_1 c_1 + c_2 s_2\right) \alpha_1 - \gamma_s \sigma^2
\]

where,

\[
F = -(\gamma_s \sigma^2 + \gamma_f \sigma^2) < 0. \\
G = (s_1 s_2 - c_1 c_2)^2 + (4 \gamma_s^2 + \gamma_f^2 + 4 \gamma_s \gamma_f) \sigma^4 + \sigma^2 (2 \gamma_s + \gamma_f) (s_1^2 + c_1^2 + s_2^2 + c_2^2) > 0 \\
H_1 = \sigma^4 \gamma_s (2 \gamma_s + \gamma_f) + (s_1 s_2 - c_1 c_2)^2 + \sigma^2 (2 \gamma_s + \gamma_f) (s_1^2 + c_2^2) + \gamma_s \sigma^2 (s_2^2 + c_1^2) > 0 \\
H_2 = \sigma^4 \gamma_s (2 \gamma_s + \gamma_f) + (s_1 s_2 - c_1 c_2)^2 + \sigma^2 (2 \gamma_s + \gamma_f) (s_2^2 + c_1^2) + \gamma_s \sigma^2 (s_1^2 + c_2^2) > 0
\]

The partial derivatives of \(\alpha_1\) and \(p_1\) with respect to \(\alpha_{1\omega}, \alpha_{2\omega}, \gamma_f, \gamma_s, s_1, s_2, c_1, c_2,\) and \(\sigma^2\) can be written
\[ \frac{\partial \alpha_1}{\partial \alpha_{1\omega}} = \frac{H_1}{G} > 0 \]
\[ \frac{\partial \alpha_1}{\partial \alpha_{2\omega}} = \frac{AF}{G} = \frac{-(\gamma_s \sigma^2 + \gamma_1 \sigma^2)A}{G} < 0 \]
\[ \frac{\partial \gamma_I}{\partial \alpha_1} = \frac{\partial \gamma_I}{\partial \alpha_1} = \frac{\partial \gamma_s}{\partial \alpha_1} = \frac{\partial \alpha_1}{\partial \alpha_1} = \frac{\partial \alpha_1}{\partial \alpha_2} = \frac{\partial \alpha_1}{\partial c_1} = \frac{\partial \alpha_1}{\partial c_2} = \frac{\partial \alpha_1}{\partial \sigma^2} = \]

and

\[ \frac{\partial p_1}{\partial \alpha_{1\omega}} = \left( s_1^2 + c_2^2 + \gamma_s \sigma^2 \right) \frac{\partial \alpha_1}{\partial \alpha_{1\omega}} - (s_1 c_1 + c_2 s_2) \frac{\partial \alpha_2}{\partial \alpha_{1\omega}} > 0 \]
\[ \frac{\partial p_1}{\partial \alpha_{2\omega}} = \left( s_1^2 + c_2^2 + \gamma_s \sigma^2 \right) \frac{\partial \alpha_1}{\partial \alpha_{2\omega}} - (s_1 c_1 + c_2 s_2) \frac{\partial \alpha_2}{\partial \alpha_{2\omega}} < 0 \]
\[ \frac{\partial p_1}{\partial \alpha_{2\omega}} \]
References


[27] Stiglitz, 1976,