Rational bubbles: an experiment[1]

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VERY VERY PRELIMINARY AND INCOMPLETE!

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Abstract

This paper studies bubbles in the laboratory. Starting with Smith, Suchanek and Williams (1988), many researchers focus on irrational bubbles. We complement this literature and design an experimental setting where bubbles can be made rational or irrational by varying one parameter. Our setting features sequential trading and sustains rational bubbles because traders are not sure to be last in the market sequence. Our analysis shows that it is pretty difficult to coordinate on rational bubbles even in an environment where irrational ones flourish.

Keywords: rationale bubbles, experiment.

1 Introduction

This paper presents an experimental investigation of speculative behavior in financial markets where bubbles both rational and irrational can arise. Recent economic developments suggest that financial markets go through periods of bubbles and crashes. The dot com mania at the turn of the last century, and the subprime mortgage frenzy are frequently interpreted as evidence that asset prices on financial markets can reach levels well above fundamental values. Likewise, Dutch Tulip, South Sea, Mississippi are names often associated with the term bubble to refer to more ancient episodes of price run ups followed by crashes. However, to the extent that fundamental values cannot be directly observed in the field, it is very difficult to empirically demonstrate that these episodes actually correspond to mispricings.

To overcome this difficulty and study bubble phenomena, economists have relied on the experimental methodology: in the laboratory, fundamental values are induced by the researchers who can then compare them to asset prices. Starting with Smith, Suchanek and Williams, 1988), many researchers document the existence of irrational bubbles in experimental financial markets. These bubbles are irrational in the sense that they would be ruled out by backward induction. The design created by Smith, Suchanek and Williams (1988) features a double auction market for an asset that pays random dividends in several successive periods. The subsequent literature shows that irrational bubbles also tend to arise in call markets (Van Boening, Williams, LaMaster, 1993), with a constant fundamental value (Noussair, Robin, Ruffieux, 2001), with lottery-like assets (Ackert, Charupat, Deaves, and Kluger, 2006), and tend to disappear when some traders are experienced (Dufwenberg, Lindqvist, and Moore, 2005), when there are futures markets (Porter and Smith, 1995) and no short-selling restrictions (Ackert, Charupat, Church, and Deaves, 2005). Lei, Noussair and Plott, 2001 modify the Smith, Suchanek and Williams, 1988 's framework to show that, even when they cannot resell and realize capital gains, some participants still buy the asset at a price which exceeds the sum of the expected dividend to be distributed, a behavior consistent with risk-loving preferences or judgmental errors.

The present paper complements this literature in three directions. First, it provides a new experimental setting where rational bubbles can be studied. Second, this experimental setting can also be used, by appropriately modifying the parameters of the experimental design, to study irrational bubbles. Having such a unified framework is useful because it allows us to compare the formation of irrational and rational bubbles. Third, it analyzes individual behavior (and not only market-level data) to try and understand the sources of individuals' speculative behavior. The objective is to enhance our understanding of the formation of bubbles in financial markets by analyzing in the laboratory both rational and irrational motives for speculation.

The design of our experimental setting is guided by two requirements: i) bubbles should be rational in some cases, and ii) the design should be simple. For bubbles to be rational, one has to prevent backward induction from operating. We obtain this property by designing a set up where participants trade a zero-value asset sequentially without knowing their position. This raises the risk of being the last to buy the asset (and not being able to sell it back). The potential capital gains offered by this asset have thus to be sufficiently high to induce risk averse traders to participate in the bubble. Simplicity is desirable so that subjects in the experiment can understand the economic situation they are involved in. It is also useful for us to interpret the data. We therefore have only two traders per market. We also impose an arbitrary price path and trade timing. This allows us to concentrate on the decision to enter into the bubble. By modifying the parameters of the price path, we can make bubbles individually rational or not.

To interpret our experimental data, we develop a theoretical analysis in line with Tirole, 1982 and Allen and Gorton, 1993 to show when a rational bubble is expected to form. Depending on the parameters of the experimental design, there are two cases. In the first case, because of backward induction, there is a unique equilibrium which is for the participants not to trade. This is related either to the fact that the price path reveals traders' position in the market process, or to the fact that some traders are too risk averse to accept to trade. In the second case, such a no-trade equilibrium still exists but there are two additional equilibria. One equilibrium is in pure strategy and consists in both traders buying the asset and riding the bubble with probability one. The other is a mixed-strategy equilibrium in which traders chose to buy the overvalued asset with a positive probability that evolves depending on the price that is proposed.

Our experimental analysis is related to Brunnermeier and Morgan, 2006 who study clock games both from a theoretical and an experimental standpoint. These clock games can indeed be viewed as metaphors of "bubble fighting" by speculators, gradually and privately informed of the fact that an asset is overvalued. Speculators do not know if others are already aware of the bubble. They have to decide when to sell the asset knowing that such a move is profitable only if enough speculators have also decided to sell. Their experimental investigation and ours share two common features: the potential payoffs are exogenously fixed (that is there is a predetermined price path), and there is a lack of common knowledge over a fundamental variable of the environment. They differ in several aspects. In Brunnermeier and Morgan, 2006, the existence of a bubble is never common knowledge (unless speculators can observe each others' moves). In our paper (except in the experimental cases where backward induction has a bite), it is traders' position in the market sequence that is never common knowledge. In addition, Brunnermeier and Morgan, 2006 focus on the timing of an exit decision while we focus on the decision to enter or not in a bubble. We decided to leave the timing issue for future research.

The experimental setting proposed in the present paper opens several avenues of research. In particular, it could be interesting to study whether the occurrence of bubbles (rational and irrational) vary with the number of traders, the introduction of risk in the underlying asset payoff, or the level of transparency (one could proxy for transparency by setting a non-null probability that a trade is publicly announced). Also, it would be interesting to extend this setting to cases where the price path and the timing are left at the discretion of traders. This would allow testing whether traders are able to coordinate on a price path that sustain rational bubbles.

2 Trading game

Our experiment focuses on the simplest financial market we can think of where bubbles can develop. We consider a financial market for an asset which only cash flow is a final payment of zero. This information is common knowledge. There are two traders on the market. Trading proceeds sequentially. Traders are assigned a position in the market sequence: each trader can be first or second with the same probability $\frac{1}{2}$. Traders do not know their position in the sequence. If they knew their position in the sequence, there would be no trade since the second trader would refuse to buy a worthless asset at a positive price. The first trader would thus also refuse to trade anticipating that the second trader would not buy the asset.

The price path is exogenous. The first trader is offered a price P_1 at which he can buy the asset. If he decides to buy the asset, he can resell it at a price P_2. For simplicity, we chose $P_2 = P_1 \times m$, where m is a positive integer. The parameter m controls the degree of explosiveness of the price path.

The price P_1 is determined as follows:

$$\tilde{P}_1 = k^{\tilde{n}},$$
$$\Pr(\tilde{n} = n) = \left(\frac{1}{2}\right)^{n+1} \text{ for all } n \in \aleph,$$

, where k is a positive integer. The resulting initial price is thus random: $\tilde{P}_1 = 1$ with probability $\frac{1}{2}$, $\tilde{P}_1 = k$ with probability $\frac{1}{4}$, $\tilde{P}_1 = k^2$ with probability $\frac{1}{8}$... When $k = m$, whatever the realization of the first price, prices which are observed do not reveal a trader's position. This is a necessary condition to prevent the operation of backward induction. The first price is indeed (i) unbounded, and (ii) any given price level $P_2 = P_1 \times m$ always belongs to the support of the random variable $\tilde{P}_1$. This second property is not true when $k \neq m$. Concerning the first property, if there was an upper bound on the first potential price, a trader being proposed a price higher than this bound would understand that he is second in the market sequence. This would turn down the possibility of a rational bubble.

In order to avoid that participants discover their position in the market sequence by hearing other traders clicking on their computers, we propose simultaneously to the first and the second trader to buy the asset. The first trader is proposed a buying price P_1 while the second one a price P_2.[1] We consider that traders are endowed with $W_0 = 1$. If they have to buy the asset for a price that is greater than one, we consider that traders can obtain funding from an outside financier, and that future cash flows are divided between the trader and the outside financier according to the proportion they initially invested to buy the asset. For example, a trader is able to buy the asset at a price of $100 by pooling his $1 with $99 from an outside financier. If the asset is then sold at a price of $1,000, the trader and the outside financier will respectively receive 1% and 99% of this amount, that is $10 and $990. Similarly, if the asset cannot be sold back, the trader and the outside financier both receive $0. These arrangements are not crucial for the underlying economics of this market since all the theoretical properties derived in the next section would hold even if traders were self-financing the entire asset acquisition. They are however very useful from an experimental point of view since they prevent us to pay potentially infinite amount of money to our subjects. Indeed, in our experiment, subjects act as traders with the outside financier contribution being automatically added to or subtracted from whatever traders invest or earn. This enables us to observe bubbles in the laboratory without having to pay potentially

[1] When the first trader does not accept to buy the asset, the second trader ends up with his initial wealth whatever his decision.

infinite amount of money to our subjects. The timing of a replication is

3 Theoretical Analysis

This section uses the environment, market mechanism and notations introduced above to derive theoretical predictions. Our analysis is based on symmetric Perfect Bayesian Nash equilibria. We focus on the case where there are only two participants on the market but identical arguments would apply with more traders.

Traders are indexed by the subscript i with $i \in \{1, 2\}$. We assume that traders' preferences can be represented by a continuous utility function $U_i\left(\tilde{W}_i\right)$, where $\tilde{W}_i$ represents his final wealth.[2] Both individuals enter the market sequentially, after being endowed with an initial wealth W_0. Each one is proposed a price P, at which he decides whether to buy the asset. If he does not trade, the agent keeps his initial wealth W_0. If he buys, he may be able to resell it at price $P_i' = m \times P_i$, in which case his final profit would be equal to: $\Pi_i = \frac{m \times P_i}{P_i} \times W_0$. However, reselling is only possible if: i) the trader is first, and ii) the second trader is willing to buy the asset at price P_i'. Otherwise, the trader looses his initial investment W_0.[3]

3.1 Traders' decision

Consider the decision tree depicted in Figure 1, and the corresponding traders' profits. When $k \neq m$, observing a price that is a not a power of k is fully informative on the trader's position in the tree. When a trader knows the node at which he stands in the tree, his decision is straightforward. No trader would indeed ever buy the asset *if he knows for sure that he is second*, since he would in this case incur a loss. By *backward induction*, the first trader should therefore not buy either: if he anticipates the second trader is not going to buy the asset, he would also incur a loss for sure by not being able to resell. These standard arguments lead to the following proposition.

Proposition 1 *For $k \neq m$, there exists a unique Nash equilibrium: no trader buys the asset.*

[2]At this point, we do not make any assumption on risk preferences.

[3]Due to the simultaneity of decisions taken by both traders in our experimental design, we propose a buy price to the second trader, even if the first one refuses to trade. In this very particular case, the second trader is prevented from incurring a loss by the first one, and he keeps his initial investment W_0.

We now focus on the case where $k = m$. In this case, traders must make a decision without knowing their position in the market sequence (unless they are proposed a price of 1, in which case they know that they are first in the sequence). Conditional on trader i being first, the other trader $(-i)$ will be second, and she will therefore observe the price $P_2 = \tilde{P}_1 \times m$. Also, conditional on trader i being second, the other trader $(-i)$ will be first, and she will observe the price $\tilde{P}_1 = \frac{P_2}{m}$. Recall that the price proposed to the first trader, $\tilde{P}_1$, is randomly chosen. For example, a price $P = k$ could be observed either by a trader entering first, with $\tilde{P}_1 = k^1$, or by a trader entering second, with $\tilde{P}_1 = k^0$. In the first case, the trader could buy the asset and potentially make a profit. In the second case, he would incur a certain loss. We denote by $x_{i,n}$ the probability with which trader i buys the asset, and by q_n the probability that he is first in the market sequence, conditional on observing a price $\tilde{P} = k^n$ (q_n does not depend on i because it is the same for both traders). His expected utility therefore writes:

$$\begin{aligned}
&EU_i\left(x_{i,n}|P = k^n\right)\\
&= q_n \times \begin{pmatrix} x_{i,n} \times \left(x_{-i,n+1} \times U_i\left(m \times W_0\right) + \left(1 - x_{-i,n+1}\right) \times U_i\left(0\right)\right) \\ + \left(1 - x_{i,n}\right) \times U_i\left(W_0\right) \end{pmatrix}\\
&+ \left(1 - q_n\right) \times \begin{pmatrix} x_{-i,n-1} \times \left(x_{i,n} \times U_i\left(0\right) + \left(1 - x_{i,n}\right) \times U_i\left(W_0\right)\right) \\ + \left(1 - x_{-i,n-1}\right) \times \left(x_{i,n} \times U_i\left(W_0\right) + \left(1 - x_{i,n}\right) \times U_i\left(W_0\right)\right) \end{pmatrix}.
\end{aligned}$$

The first parenthesis corresponds to the case where trader i turns out to be first in the market sequence. Trader i accepts to buy with probability $x_{i,n}$ and receives a payoff that depends on the other trader's decision. When the other trader buys, which occurs with probability $x_{-i,n+1}$, trader i's payoff is $m \times W_0$. When the other trader does not buy, which occurs with probability $(1 - x_{-i,n+1})$, trader i's payoff in this case is 0. The same logic applies to the case where trader i is second in the market process. A strategy profile in our setting is defined by $(x_{i,n}, x_{-i,n}), \forall n$.

Given the distribution of initial prices $\tilde{P}_1$ indicated in the previous section, we can compute q_n, for all $n \in \aleph$, that is the probability to be the first trader conditional on observing $\tilde{P} = k^n$. This probability is given in the following Lemma.

Lemma 1 *For $k = m$, then:*

$$\forall i, \forall n \geq 1, q_n \equiv \Pr\left(1st|\tilde{P} = k^n\right) = \frac{1}{3}$$

$$q_{i,0} \equiv \Pr\left(1st|\tilde{P} = k^0\right) = 1$$

This Lemma shows that for $n \geq 1$, the probability to be the second trader is constant: high prices are not more likely than a low price to signal a late arrival in the sequence of traders. A particular case arises when a trader observes the price $\tilde{P} = 1$. In this case indeed, the trader can infer his position as first trader from his observation. Still, what matters is that the second trader cannot infer from observing a price $P = k$ whether he is first (with $P_1 = k^1$), or second (with $P_1 = k^0$).

3.2 The equilibria

We now solve for the equilibria of this game. Consider first the case where trader i observes $P = 1$ (that is, $n = 0$). He perfectly infers his position as first trader. Let us define:

$$\Delta_i \equiv \frac{U_i\left(m \times W_0\right) - U_i\left(0\right)}{U_i\left(W_0\right) - U_i\left(0\right)}.$$

Δ_i depends on the curvature of the trader's utility function and decreases with risk aversion. Comparing his expected utility when he buys or not yields:

$$EU_i\left(x_{i,0} = 1\right) > EU_i\left(x_{i,0} = 0\right) \Leftrightarrow x_{-i,1} > \frac{1}{\Delta_i} \tag{1}$$

Even if trader i knows that he is first in the sequence, his decision depends on the strategy of the other trader, $-i$, when she observes a one-step ahead price, $P = k^0 \times m$.

Consider now the case where trader i observes $P = k^n$, with $n \geq 1$. He then expects to be the first trader only with probability q_n. Consequently:

$$\forall n \geq 1, EU_i\left(x_{i,n} = 1\right) > EU_i\left(x_{i,n} = 0\right) \Leftrightarrow x_{-i,n+1} > \frac{1}{\Delta_i} \times \left(1 + \frac{1 - q_n}{q_n} \times x_{-i,n-1}\right) \tag{2}$$

Compared to the previous case, trader i's decision not only depends on the strategy of the other trader $-i$ when she observes a one-step ahead price, $P = k^n \times m$, but also on her decision when she observes a one-step below price, $P = \frac{k^n}{m}$. Assume for instance that she never buys when she observes a price $P = k^{n-1}$. In this case, when trader i observes the price $P = k^n$, either he is first, or he is second but does not make a loss even when he decides to buy since he is "protected" by trader $-i$ who has not bought.

Conditions (1) and (2) first show that no trader would ever buy if he expects the other trader not to buy with probability one. The following proposition follows.

Proposition 2 *Whatever traders' preferences, there always exists a Nash equilibrium (in pure strategies) such that no trader buys the asset, whatever the price he observes.*

What we are interested in, though, is whether an equilibrium characterized by the presence of a speculative bubble, could occur in this market. Notice that for $m > 1$, whatever the agents' preferences, the following inequality holds:

$$0 < \frac{1}{\Delta_i} < 1$$

Consequently, for $n = 0$, Condition (1) shows that the optimal strategy of trader i observing $P = 1$ is to buy the asset, provided that the other trader buys with probability one when he observes a price $P = k^1$. For $n \geq 1$ though, his optimal decision when observing $P = k^n$ also depends (i) on the behavior of the other trader when she observes the one-step below price $P = k^{n-1}$, and (ii) on the probability to be the first trader, q_n. To better understand the impact of this probability q_n, let us try to determine whether there exists an equilibrium in pure strategies, such that both traders buy the asset.

Assume that trader $-i$ always buys the asset whatever the price she observes. Then as seen above, trader i would also buy the asset when he observes $P = 1$. When i observes a price $P = k^n$ for $n \geq 1$ though, his reaction depends on his preferences. In this case indeed, Condition (2) implies that he buys if and only if $1 \leq q_n \times \Delta_i$. If trader i is highly risk adverse (that is, if Δ_i is low), he would never buy the asset. The opposite holds. The existence of an equilibrium in pure strategies, characterized by the entry of both traders with probability one, therefore relies on conditions regarding traders' preferences. When these conditions are satisfied, there also exist equilibria in mixed strategies. These results are stated in the following proposition.

Proposition 3 *A) For $k = m$, if $\Delta_i \times q_n \geq 1$ for $i \in \{1, 2\}$, then there exist three sets of Nash equilibria:*
i) an equilibrium in pure strategies with no bubble: $\forall n \geq 0, x^*_{i,n} = 0$,
ii) an equilibrium in pure strategies with a bubble: $\forall n \geq 0, x^*_{i,n} = 1$,
iii) a multiplicity of equilibria in mixed strategies characterized by $\forall n \geq 4, x^*_{i,n} = \frac{1}{\Delta_{-i}} \times \left(1 + \frac{1-q_n}{q_n} x^*_{i,n-2}\right)$.
Strategies $x_{i,n}$ for $n < 4$ are described in the Appendix.
B) For $k = m$, if $\Delta_i \times q_n < 1$ or $\Delta_{-i} \times q_n < 1$, there exists a unique equilibrium such that no trader buys the asset, whatever the price he observes: : $\forall n \geq 0, x^*_{i,n} = 0$.

When they exist, the mixed-strategy equilibria all converge (in n) to $x_{i,n} \mapsto x_i^* = \dfrac{1}{\Delta_{-i} - \frac{1-q_n}{q_n}}$. This convergence imply a probability of buying the asset that increases or decreases with the proposed price depending on the value of the preference parameters as well as on the initial probabilities $x_{i,0}$ and $x_{i,2}$ traders coordinate on.

4 Experimental Design and Hypotheses

Our experiment includes a total of 92 subjects. Subjects are first-year students in the Master in Finance and the Master in International Management at the University of Toulouse. We have six sessions with 12 to 18 subjects per session. Each subject participates in only one session. Each session includes ten replications of the trading game. Several weeks before the market sessions are run, subjects' risk aversion is measured thank to Laury and Holt (2002)'s procedure. At each replication of the trading game, subjects are randomly paired in a complete stranger design, that is, a subject does not play twice with the same person. The experiment is computerized and programmed with z-tree (see Fischbacher, 2007 for a presentation of this software). For sessions 1 and 4, $k = m = 3$. For sessions 2 and 3, $k = m = 10$ and $k = m = 20$, respectively. For session 5, $k = 10$ and $m = 11$. For session 6, $k = 20$ and $m = 21$. The following predictions are derived from the theoretical analysis presented in the previous section. In particular, Proposition 1 yields:

Hypothesis 1: In the sessions where $k = 10$ and $m = 11$, and where $k = 20$ and $m = 21$, we should observe no bubble.

Given that Δ_i increases with m, Proposition 3 yields:

Hypothesis 2: Bubbles should be more frequent and the probability to buy should be larger in the session where $k = m = 20$ than in the session where $k = m = 10$, which should be larger than in the session where $k = m = 3$.

Hypothesis 3: Bubbles should not occur in the session where $k = m = 3$, unless traders are risk neutral or risk lovers.

Proposition 3 suggests that there is multiplicity of equilibria:

Hypothesis 4: When $k = m = 10$ and $k = m = 20$, there is a coordination problem since *traders have to choose between playing either the bubble equilibrium or the no-trade equilibrium (or even the mixed-strategy equilibria).*

As became apparent in the above predictions, we have chosen several values for k and m such that bubbles are rational in some sessions and irrational in others. Furthermore, the reason why bubbles are irrational varies across sessions. In sessions 2 and 3, for reasonable levels of risk aversion, it can be rational for traders to enter into the bubble. In sessions 1 and 4, the price path does not reveal to traders their position in the market sequence. However, the payoff to be earned in case of a successful buying decision is not high enough to induce a risk averse trader to enter in the bubble. The bubble is thus rational only if traders are risk neutral or risk lovers and this fact is common knowledge. In sessions 5 and 6, bubbles cannot be rational because prices reveal traders' position in the market process.

Our experimental design is summarized in Table 1.

TABLE 1

Session	Number of Subjects	k	m	Bubble
1	16	3	3	Rational only if risk neutrality
2	16	10	10	Rational
3	16	10	11	Irrational
4	18	3	3	Rational only if risk neutrality
5	12	20	20	Rational
6	14	20	21	Irrational

5 Results

We first start by analyzing overall market behavior. At this aggregate level, we can measure the frequency as well as the magnitude of bubbles. The frequency of bubbles is defined as the proportion of replications where at least one of the traders accepted to buy the asset. The magnitude of bubbles is referred to as large if both traders accepted to buy the asset, and small if only one trader accepted to buy the asset. Figure 2 presents the results per session for the overall ten replications of the experiment. Session 1 and 4 are pooled because they use the same parameter values. Figure 3 differentiates between the first and the last five replications.

Figure 2 shows that, when k and m are equal and increase, we have more and more large bubbles. This is consistent with our hypothesis 2. This suggests that rational large bubbles are more and more frequent when the price path becomes more and more explosive. This result also holds for small bubbles except that there is no difference between the cases in which $k = m = 10$ and $k = m = 20$. There is a quite high proportion of replications without a bubble when $k = m = 10$ and $k = m = 20$. This indicate that traders fail to perfectly coordinate on the bubble equilibrium and is consistent with hypothesis 4. The fact that there is a significant number of bubbles in the case in which $k = m = 3$ suggests that traders are eager to speculate even if the potential gains are limited. Given that our measure of risk preferences indicates that most of our traders are risk averse, we are incline to interpret those bubbles as irrational. Figure 2 also shows that there are bubbles in an environment where backward induction is supposed to shut down speculation, namely when $k \neq m$. This is in line with the previous experimental literature cited in the introduction and contradicts our hypothesis 1. More surprisingly, we observe that the frequency of irrational bubbles also increase when k and m increase. It is possible that subjects, because they know that they are going to play several replications of the game, adopt a reciprocal behavior: they enter in the bubble even if they know that they are second and expect others to do so. An alternative explanation is just that they have social preferences and are happy to loose a little to make someone else profit a lot from the bubble. A last interpretation is related to bounded rationality. It is possible that some traders are making mistakes and buy when last in the market sequence. If the first trader in the sequence anticipates this behavior, it may be optimal for him to buy the asset. Figure 3 gives us more information on these issues. It shows that there tend to be less bubbles (both rational and irrational) during the last five than the first five replications. This suggests that the coordination problems might prevent the formation of rational bubbles and that learning might help traders avoid making damageable decisions.

To gain more insight on the formation of bubbles, we now study individual decisions, that is whether or not traders buy the overvalued asset. We focus on several variables: the unconditional probability to buy, the probability to buy conditional on the trader knowing that he is first, the probability to buy conditional on the trader knowing that he is second, the probability to buy conditional on the trader ignoring his position. Figure 4 reports the average across the ten replications of the experiment while Figure 5 separates the first and the last five replications.

Figure 4 shows that the unconditional probability to buy the overvalued asset increases with the potential profit. This is true for all sessions and is in line with the previous results on the occurrence of bubbles. The probability to buy the asset given that a trader knows that he is first is very high when $k = m$. This indicates that when they know that they are at the beginning of the market sequence, traders tend to coordinate on the bubble equilibrium. The probability to buy when a trader ignores his position is significantly lower than when he knows that he is first. This is consistent both with the fact that he faces a more risky decision and with the fact that traders are reluctant to coordinate on the bubble equilibrium. Regarding the sessions where $k \neq m$, it appears that the tendency to buy the asset increases with the explosiveness of the price path whether the trader knows he is first or second. This parallel relationship is in line with the bounded rationality interpretation that we developed above. Figure 5 shows i) that these results are robust for both the first and the last five replications, and ii) that the propensity to enter into a bubble does not increase over time.

In order to complement these results on individual behavior, we run a logit panel regression that tries and explain the propensity to buy the overvalued asset as a function of several variables: the level of the subject's risk aversion, a dummy variable indicating that the subject knows he is first, a dummy variable indicating that the subject knows he is second, a dummy variable indicating that the subject ignores his position, the explosiveness of the price path (m), a dummy variable indicating that $k \neq m$, the percentage of the time where the subject has been confronted in the past with a trader that decided to buy (PBuy), and the proposed price. A logit regression is adequate because we try to explain a dummy variable: either the subject chooses to buy the asset (we code 1) or not (we code 0). Panel corrections are also warranted because the same subject participates in several replications. The results are in Table 3 and show that, apart from the level of risk aversion and the PBuy variable, all the explanatory variables have statistically significant coefficients. The sign of these coefficients is intuitive: a subject's propensity to buy the overvalued asset increases when he knows he is first in the market sequence, decreases when he knows he is second, increases with the explosiveness of the price path, decreases when $k \neq m$, and decreases with the proposed price.

6 Appendix

Proof. Proposition 1

Given the distribution of $\tilde{P}_1$, notice that for $k \neq m$, the second trader will exactly know his position of last trader in the market sequence. The second trader will never buy the asset, whatever his utility function. By backward induction, the first trader will not buy the asset either. ■

Proof. Lemma 1

$$\forall n \geq 1, q_n \equiv \Pr\left(i = 1\text{st}|\tilde{P} = k^n\right) = \frac{\Pr\left(i = 1\text{st} \cap \tilde{P}_i = k^n\right)}{\Pr\left(\tilde{P}_i = k^n\right)}$$
$$= \frac{\Pr\left(\tilde{P}_i = k^n|i = 1\text{st}\right) \times \Pr(i = 1\text{st})}{\Pr\left(\tilde{P}_i = k^n|i = 2\text{nd}\right) \times \Pr(i = 2\text{nd}) + \Pr\left(\tilde{P}_i = k^n|i = 1\text{st}\right) \times \Pr(i = 1\text{st})}$$
$$= \frac{\Pr\left(\tilde{P}_1 = k^n\right) \times \frac{1}{2}}{\Pr\left(\tilde{P}_1 = k^{n-1}\right) \times \frac{1}{2} + \Pr\left(\tilde{P}_1 = k^n\right) \times \frac{1}{2}}$$
$$= \frac{\frac{1}{2^{n+1}}}{\frac{1}{2^n} + \frac{1}{2^{n+1}}}$$
$$= \frac{1}{3}$$

■

Proof. Proposition 2

It is straightforward, considering Conditions (1) and (2) for $i = \{1, 2\}$, to show that, for all n and i, $x^*_{i,n} = 0$ is a Nash Equilibrium. ■

Proof. Proposition 3, case B) where $\Delta_i < \dfrac{1}{q_n}$ or $\Delta_{-i} < \dfrac{1}{q_n}$

Consider the case where $\Delta_i < \dfrac{1}{q_n}$. There exists an equilibrium in mixed strategies if, for $n \geq \underline{n}$, agent i is indifferent between buying or not. Condition (2) becomes:

$$\forall n \geq \underline{n}, x_{B,n+1} = \frac{1}{\Delta_i} \times \left(1 + \frac{1 - q_n}{q_n} \times x_{-i,n-1}\right)$$

We define the indifference curve function $IC_{i,n}$ as follows:

$$IC_{i,n}(x) = \frac{1}{\Delta_i} \times \left(1 + \frac{1 - q_n}{q_n} \times x\right)$$

An equilibrium must be such that $\forall n \geq \underline{n}$, $IC_{i,n}(x_{-i,n-1}) = x_{-i,n+1}$. Figure A1 represents trader i's indifference curve between buying or not, when he observes $n \geq \underline{n}$. For $\Delta_i < \dfrac{1}{q_n}$, it can be easily shown that this indifference curve never crosses the line $y = x$, whether the slope of the indifference curve is lower or larger than 1. In this case, it will always be possible to find a $n^* > \underline{n}$ such that, at x_{-i,n^*-1}, the condition of the indifference curve yields to a value $IC_{i,n}\left(x_{-i,n^*-1}\right)$ that would be strictly greater than 1. As a consequence, the following inequality holds:

$$\frac{1}{\Delta_i} \times \left(1 + \frac{1-q_n}{q_n} \times x_{-i,n^*-1}\right) > 1 \geq x_{-i,n^*+1}.$$

The best response of trader i when he observes a price $P = k^{n^*}$ is therefore not to buy the asset, yielding $x_{i,n^*} = 0$. i is no more indifferent between buying or not. By backward induction, this dynamically rules out any equilibrium where traders buy the asset with a positive probability for any price observed. The reasoning is similar for $\Delta_i < \frac{1}{q_n}$. ■

Proof. Proposition 3, case A where $\Delta_A \geq \frac{1}{q_n}$ and $\Delta_B \geq \frac{1}{q_n}$. The proofs of i) and ii) are straightforward and thus omitted.

Let us show that there exists a multiplicity of equilibria in mixed strategies characterized, for $i = 1, 2$, by:

$$\forall n \geq 4, x_{i,n+2} = \frac{1}{\Delta_{-i}} \times \left(1 + \frac{1-q_n}{q_n} x_{i,n}\right).$$

and i) either

$$\begin{aligned}
x_{i,0} &= x_{i,1} = 1, \\
x_{i,2} &\geq \frac{1}{\Delta_{-i} \times q_n}, \\
x_{i,3} &= \frac{1}{\Delta_{-i} \times q_n},
\end{aligned}$$

, or ii)

$$\begin{aligned}
x_{i,0} &= x_{i,1} = 0, \\
x_{i,2} &< \frac{1}{\Delta_{-i}}, \\
x_{i,3} &= \frac{1}{\Delta_{-i} \times q_n},
\end{aligned}$$

, or iii)

$$\begin{aligned}
x_{i,0} &\in]0, 1[, \\
x_{i,1} &= \frac{1}{\Delta_{-i}}, \\
x_{i,2} &= \frac{1}{\Delta_{-i}} \times \left(1 + \frac{1-q_n}{q_n} x_{i,0}\right).
\end{aligned}$$

Figure A2 represents trader i's indifference curve between buying or not, when he observes $n \geq 1$. For $\Delta_i \geq \frac{1}{q_n}$, it can be easily shown that this indifference curve crosses the line $y = x$ within the $(1,1)$ square. We define this crossing point between the line $IC_{i,n}$ and $y = x$ as x^*_{-i} and show that: $x^*_{-i} = \frac{1}{\Delta_i - \frac{1-q_n}{q_n}}$.

- Assume that the point $(x_{-i,0}, x_{-i,2})$ stands strictly *below* trader i's indifference curve when he observes $n = 1$, but is such that $x_{-i,0} > 0$. i's best response is then $x_{i,1} = 0$ (Condition (2) for i and $n = 1$). This contradicts our assumption $x_{-i,0} > 0$ since for $x_{i,1} = 0$, condition (1) shows that $-i$'s best response for $n = 0$ is $x_{-i,0} = 0$.

- Assume that the point $(x_{-i,0}, x_{-i,2})$ stands strictly *above* trader's A indifference curve when he observes $n = 1$, but is such that $x_{-i,0} < 1$. i's best response is then $x_{i,1} = 1$ (Condition (2) for i and $n = 1$). This contradicts our assumption $x_{-i,0} < 1$ since for $x_{-i,1} = 1$, condition (1) shows that $-i$'s best response for $n = 0$ is $x_{-i,0} = 1$.

- Assume that the point $(x_{-i,0}, x_{-i,2})$ stands on the grey line in Figure A2, i.e.:

  - either *on* trader i's indifference curve when he observes $n = 1$, but is such that $x_{-i,0} > 0$ and $x_{-i,0} < 1$. Then Condition (1) shows that to avoid a contradiction in $-i$'s best response when she observes $n = 0$, it must be the case that $x_{i,1} = \frac{1}{\Delta_{-i}}$.
  - or strictly *above* trader i's indifference curve when he observes $n = 1$, but is such that $x_{-i,0} = 1$. According to our definition, we have $x'_{-i} < x_{-i,2} \leq 1$. i's best response is then $x_{i,1} = 1$ (Condition (2) for i and $n = 1$), which does not contradict our assumption $x_{-i,0} = 1$, as shown in condition (1).
  - or strictly *below* trader's i indifference curve when he observes $n = 1$, but is such that $x_{-i,0} = 0$.

  Then conditions for $n = 0, 1$ and 2 described above ensure that neither Condition (1) nor (1) leads to a contradiction. Intuitively, wherever the initial strategies of traders when they observe $n = 0$ and $n = 2$ on this grey line, there will always be a n^{**} (either 1 or 2) such that the couple $(x_{i,n^{**}}, x_{i,n^{**}+2})$ stands on the other trader's indifference curve when she observes $n = n^{**} + 1$. Once this is the case, there exists an equilibrium which converges to x^*_i, as shown graphically in Figure A3.

■

7 References

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Figure 1 Decision Tree

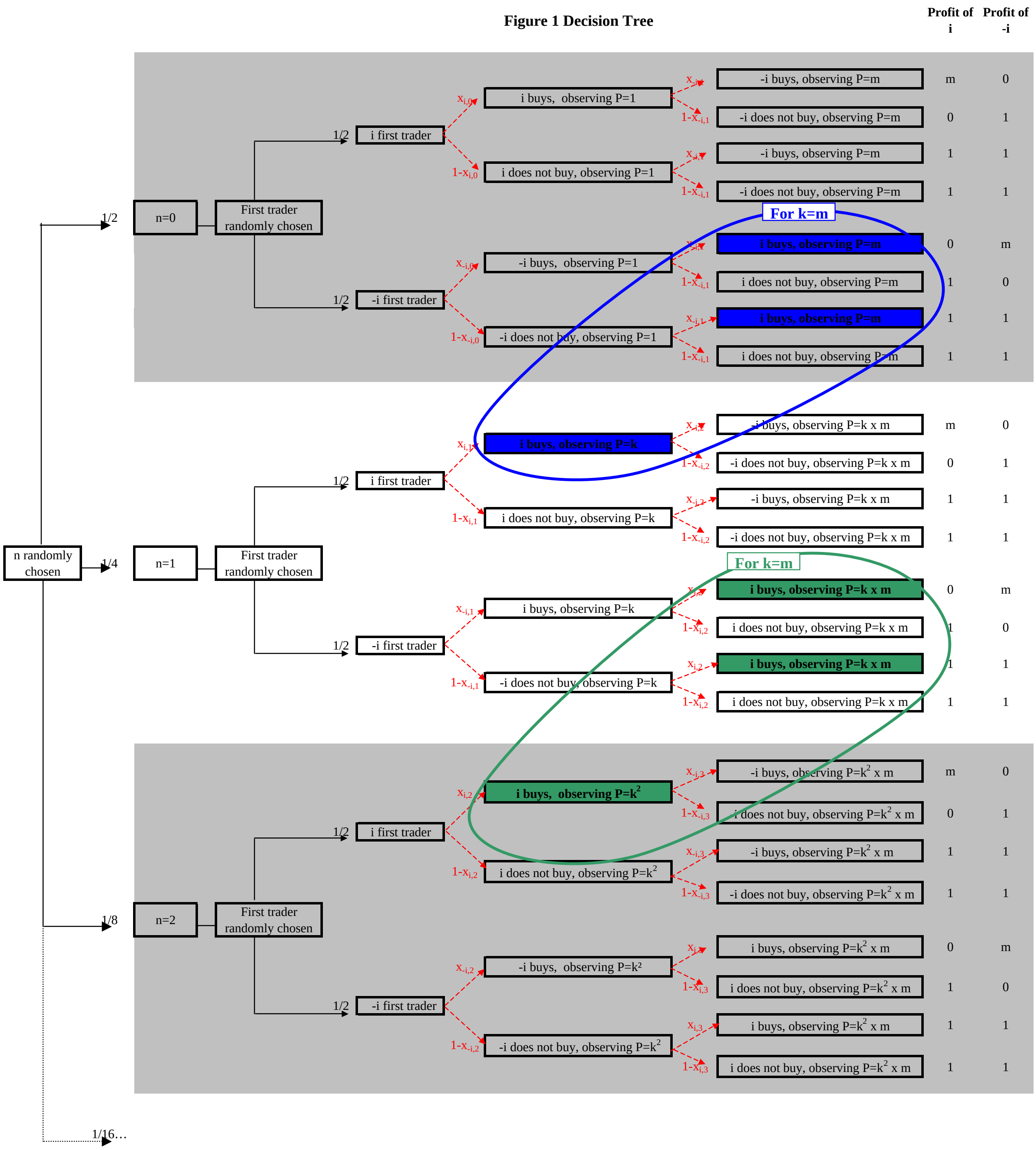

Figure 2: Probability to observe bubbles, per session

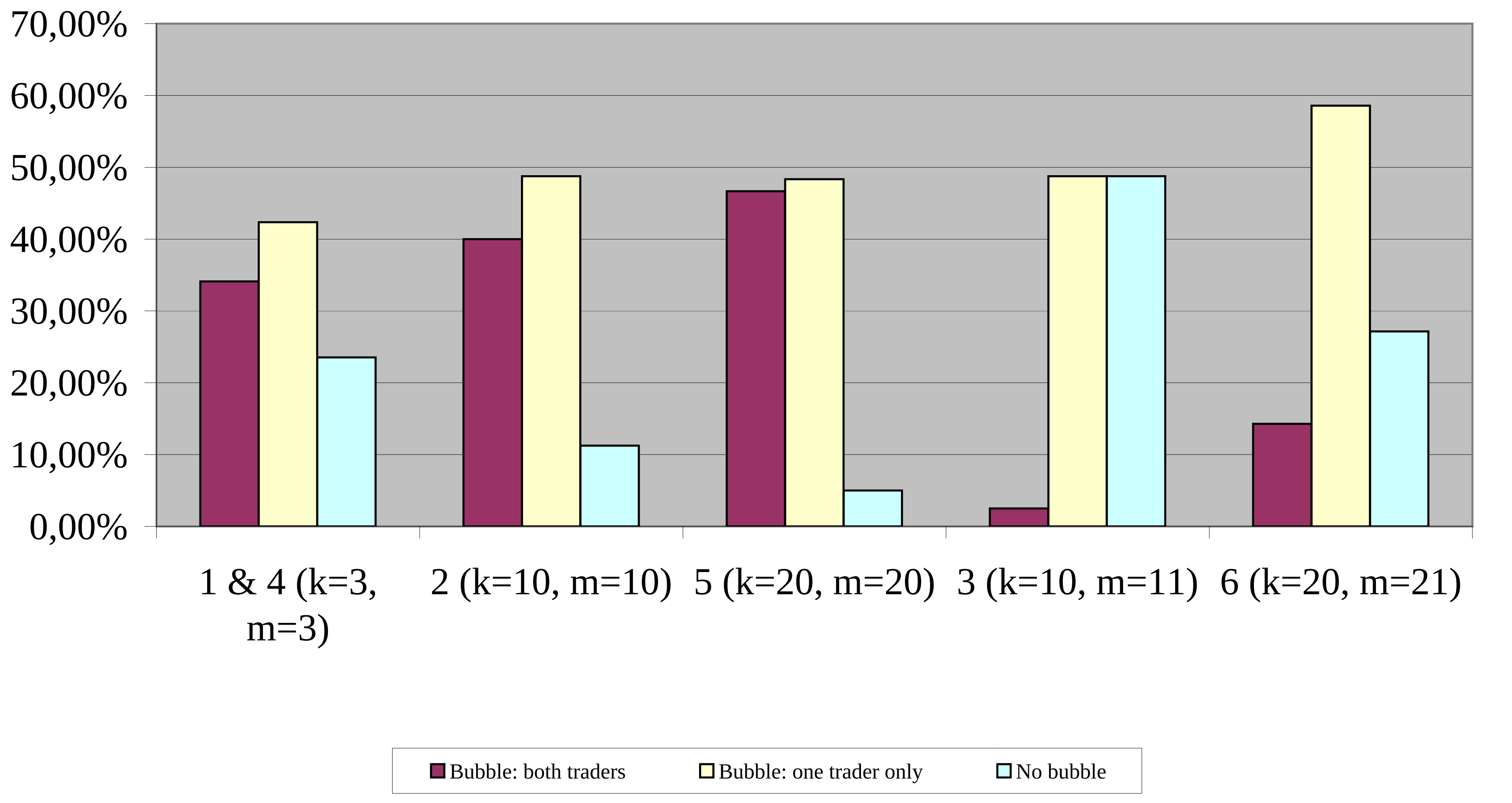

Figure 3: Probability to observe bubbles per session: First 5 periods vs Last 5 periods

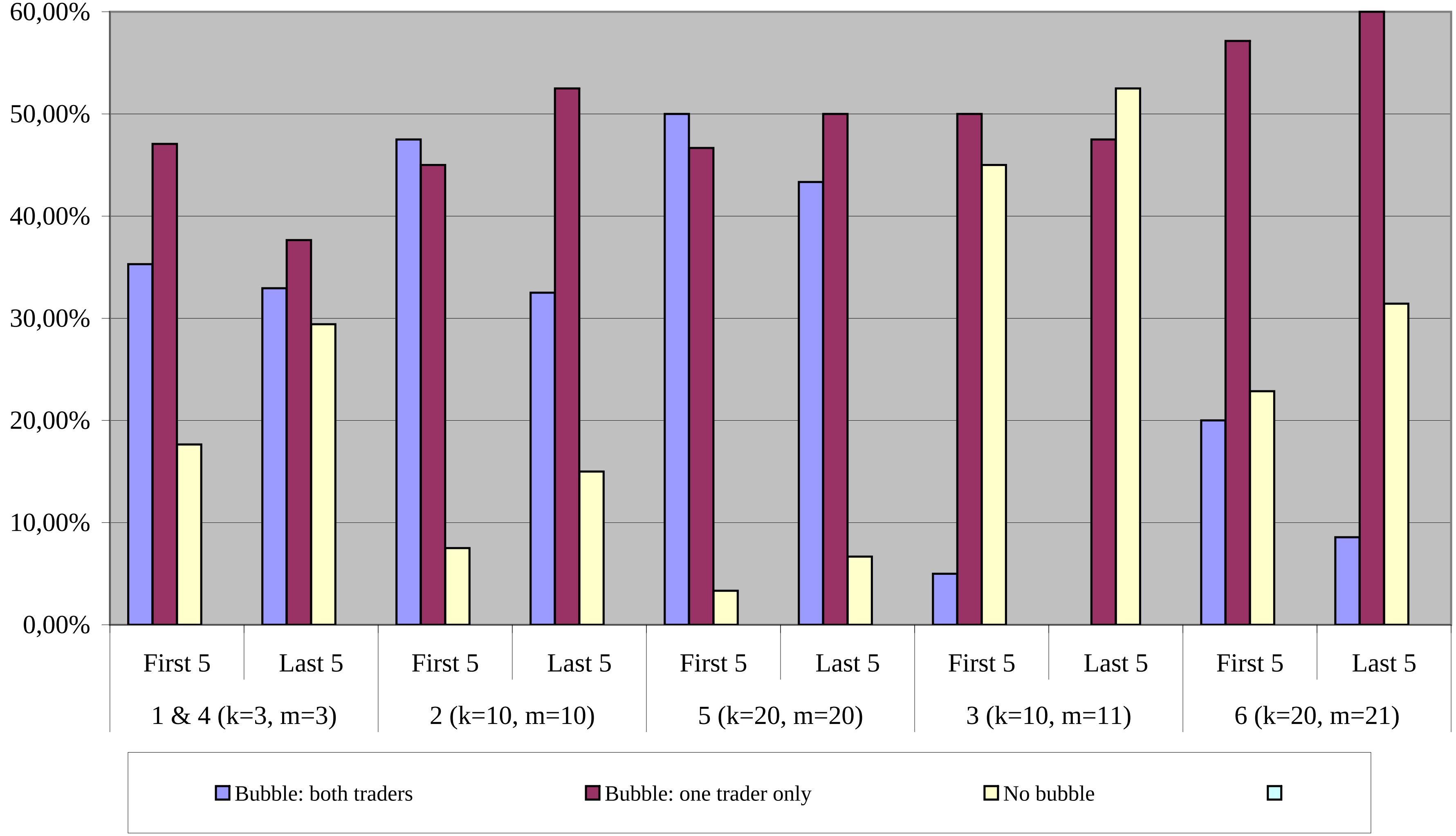

Figure 4: Entry decision, conditional on trader's inferences

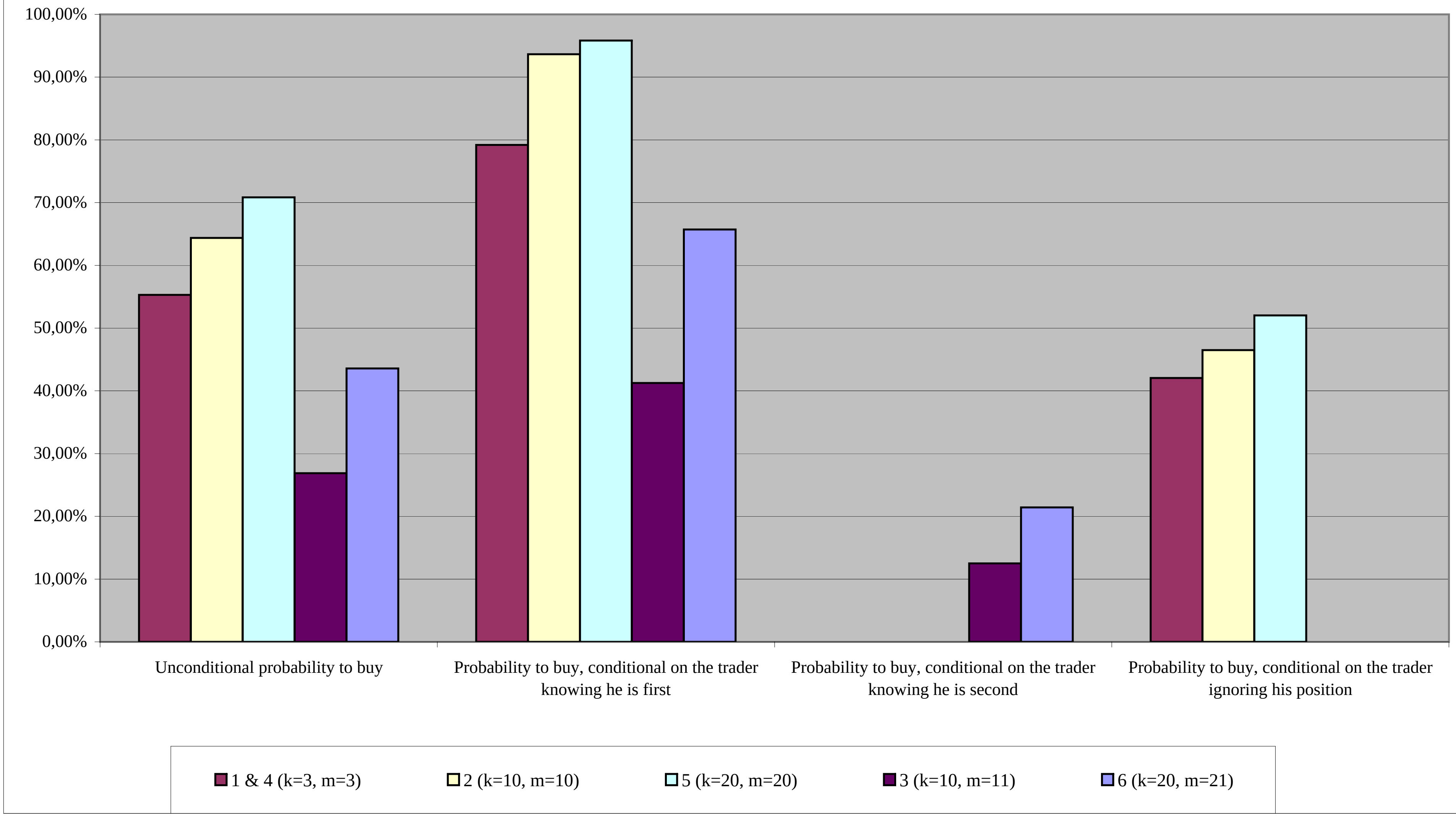

Figure 5: Entry decision per session: First 5 periods vs Last 5 periods

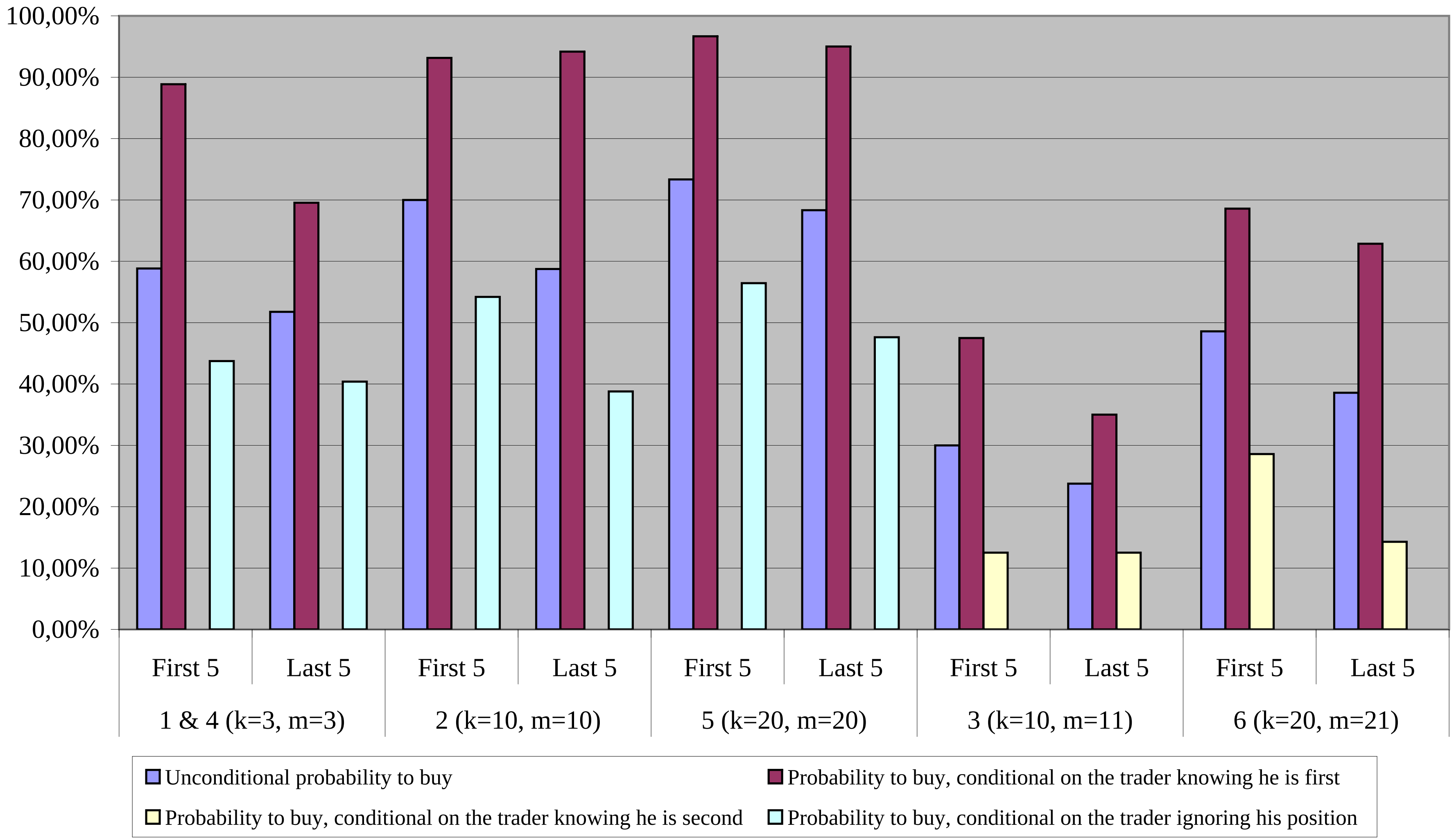

Table 2 **Non-parametric tests of equality of medians across sessions**

	Conditional and unconditional probabilities to buy				Probability of occurrence of a bubble		
	Unconditional probability to buy	Probability to buy, conditional on the trader knowing he is first	Probability to buy, conditional on the trader knowing he is second	Probability to buy, conditional on the trader ignoring his position	Bubble: both traders	Bubble: one trader only	No bubble
Session 1&4 (k=3, m=3) vs. Session 2 (k=10, m=10)	-1.529 / 0.1262	-1.869 / 0.0616		-0.647 / 0.5176	-0.862/0.3888	-0.891/ 0.3729	1.592/ 0.1115
Session 1&4 (k=3, m=3) vs. Session 3 (k=10, m=11)	3.155 / 0.0016	3.159 / 0.0016			3.912/ 0.0001	-0.770/0.4411	-2.924/ 0.0035
Session 1&4 (k=3, m=3) vs. Session 5 (k=20, m=20)	-1.971/ 0.0488	-1.776 / 0.0758		-0.540 / 0.5893	-1.540/ 0.1235	-0.641/0.5212	2.724/0.0064
Session 1&4 (k=3, m=3) vs. Session 6 (k=20, m=21)	1.446 / 0.1482	1.654 / 0.0980			2.668/0.0076	-1.817/0.0692	-1.178/0.2389
Session 2 (k=10, m=10) vs. Session 3 (k=10, m=11)	3.418 / 0.0006	3.443 / 0.0006			3.483/0.0005	-0.055/0.9561	-3.403/0.0007
Session 2 (k=10, m=10) vs. Session 5 (k=20, m=20)	-0.874 / 0.3823	0.229 / 0.8188		-0.047 / 0.9628	-0.987/ 0.3237	0.338/0.7354	1.239/0.2153
Session 2 (k=10, m=10) vs. Session 6 (k=20, m=21)	2.472 / 0.0134	2.815 / 0.0049			3.155/0.0016	-1.353/0.1762	-2.524/0.0116
Session 3 (k=10, m=11) vs. Session 5 (k=20, m=20)	-3.473 / 0.0005	-3.386 / 0.0007			-3.248/0.0012	0.131/0.8957	3.137/0.0017
Session 3 (k=10, m=11) vs. Session 6 (k=20, m=21)	-1.705 / 0.0882	-1.461 / 0.1441	-1.562 / 0.1183		-2.981/0.0029	-1.073/0.2834	2.486/0.0129
Session 5 (k=20, m=20) vs. Session 6 (k=20, m=21)	2.597 / 0.0094	2.803 / 0.0051			2.887/0.0039	-1.321/ 0.1866	-3.082/0.0021

Table 3   Logit Panel Regression with Random Effects on the Buy Decision

	Coefficient	Statistic	p-value
Relative risk aversion coefficient	0,35	0,75	0,4510
Offered buy price	-0,00	-1,48	0,1400
Dummy which takes the value 1 if the trader knows he is first, and 0 otherwise	1,52	2,48	0,0130
Dummy which takes the value 1 if the trader knows he is second, and 0 otherwise	-1,30	-1,67	0,0950
Dummy which takes the value 1 if the trader ignores his position, and 0 otherwise	-1,62	-2,74	0,0060
Explosiveness of the price path (m)	0,09	2,39	0,0170
Dummy which takes the value 1 if parameters k and m differ, and 0 otherwise	-3,22	-5,12	0,0000
Frequency which with the trader has met other buyers	0,41	0,75	0,4520
Log likelihood	-349,1033		
Prob > chi2	0,0000		

Figure A1 : $\Delta_i < 1/q_n$

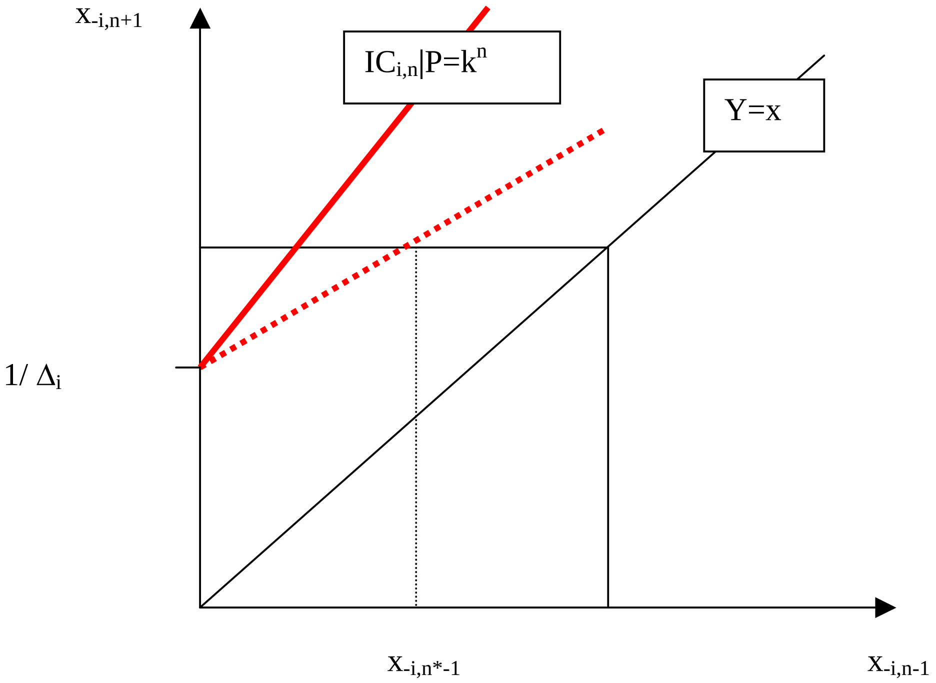

Figure A2 : $\Delta_i >= 1/q_n$

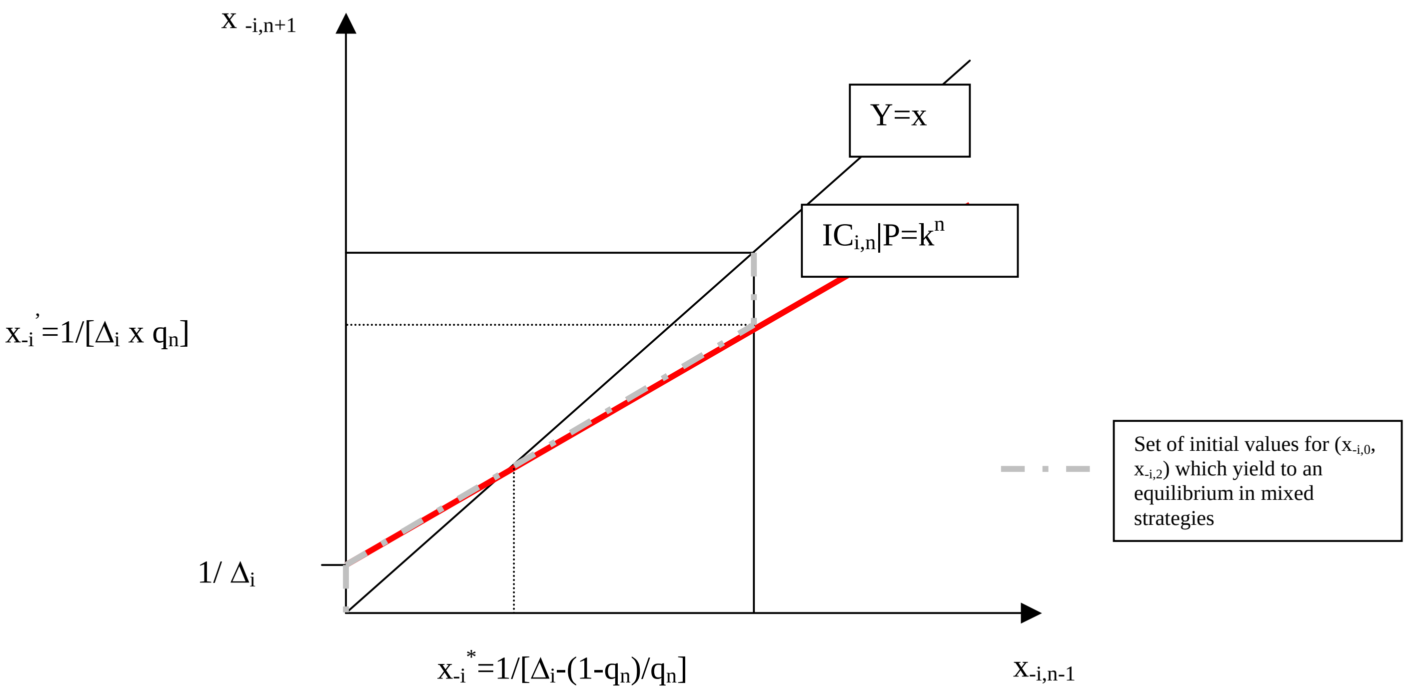

Figure A3 : $\Delta_i >= 1/q_n$